Natural frequency of Cobiax® flat slabs

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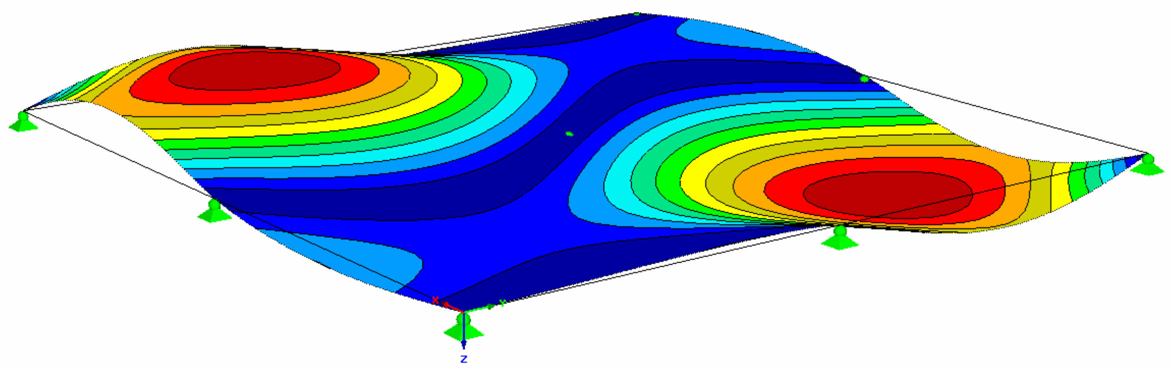

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Abstract

The problem of human discomfort due to low-level vibrations of concrete slabs is an important factor of consideration during any design process. The continuing trend towards large open floors, free of partitions, and increased slenderness in design aesthetics increases the likelihood of annoying floor vibration induced by small impacts such as human footfalls.

The present research covers several areas concerned with addressing this problem. A basic literature review of previous work in the field of floor vibrations is presented and provides an introduction into this general topic.

At first some recommendations of acceptance limits by national standards as well as by independent authors are presented. The problems hindering the proper evaluation of floor vibration are also shown.

The next area of study involves simplified hand calculation methods for the approximate estimation of concrete slabs' fundamental frequencies. Different approaches are presented and afterwards compared and evaluated against several example solutions from accurate finite element software.

The third chapter focuses on the fundamental frequency of a specific biaxial hollow concrete slab: the Cobiax flat slab. An investigation of its vibration behaviour under different parameters was carried out using finite element software. The data of seven different floor designs was obtained and compared to conventional solid slabs, leading to a final evaluation of the vibration performance of Cobiax flat slabs.

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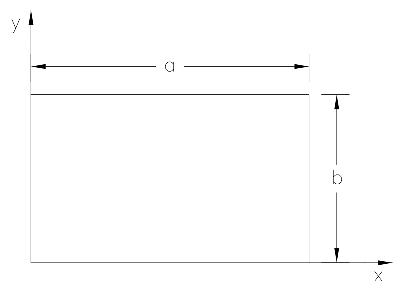
Notation

a	Length of plate
b	Width of plate
C	Clamped edge
CS	Cobiax flat slab
D	Plate rigidity
E	Young's modulus
f	Natural frequency
f_0	Fundamental frequency
F	Free edge
g	Gravity
G	Modulus of rigidity
h	Plate thickness
I	Moment of inertia
I_x	Moment of inertia in x-direction
I_y	Moment of inertia in y-direction
k	Stiffness
m	Mass (per unit area)
S	Simply supported
SS	Solid slab
α	Length-width ratio
β	Damping ratio
γ	Density of plate material
ν	Poisson's ratio
ω	Circular frequency (= 2π x frequency)



Contents

Figures

Tables

1. Introduction and Background Knowledge

1.1 Introduction

In the last years, the number of floor vibration complaints in residential buildings and offices increased significantly (Hanagan 2005, Williams and Waldron 1994, Naeim F. 1991). The two usual causes for this annoying problem are human activities such as walking, running, jumping or dancing and mechanical movement from, for example, air-conditioning systems, heating, and washing and drying machines. In rarer cases, indirect excitations from automobiles on parking levels below a floor, or transmitted vibration through building columns from other floors or the ground are to blame.

The psychological effect of the up-and-down motion caused by floor vibration can be immense. Generally, it gives people an ‘unpleasant’ feeling and prompts fear of structural collapse. This feeling increases even more if a person is not actively involved in inducing the acting load. People’s quality of life and working conditions, then, are negatively affected by perceptible vibration and so it is usually considered undesirable.

However, floor vibration does not only affect the inhabitants of a building; in extreme cases it can also lead to fatigue failures or damage structural elements which results in costly remodelling. Additionally, buildings housing sensitive equipment such as hospitals, laboratories and manufacturing plants that use modern micro- and nanotechnologies are in especial need of protection.

The problem of vexatious floor vibration is not new. Civil engineer Thomas Tredgold (1828) wrote: "girders should always be made as deep as they can to avoid the inconvenience of not being able to move on the floor without shaking everything in the room." In the past, a simple deflection criterion (deflection of less than span/x under distributed live load) usually ensured structures against ‘heavy’ vibration, but because of the current trend towards longer spans and lighter floor systems (the result of more aesthetical and efficient constructions), this approach no longer works and the need to reconsider floor vibration has increased. Slender structural forms and decreased floor mass reduce natural frequency as well as structural damping and so floor vibration has become an area of concern.

There exist several ways to prevent or at least reduce this problem. The simplest and most effective method for machinery-induced floor vibration is to isolate the source from the ground. This could be by means of springs, insulating plates or other elastic bodies.

However, for human-induced vibration it is impossible to isolate the source from the floor system. In this case humans are both the source and receiver of vibrations which makes the situation very difficult. Thus, the structure itself must be considered and modified to prevent annoying floor vibration. One way of addressing this problem is to increase natural frequency to a level which can hardly be perceived by a building's occupants.

1.2 Natural Frequency

Natural frequency is one of the fundamental parameters used in the determination of a structure's response to dynamic loads. It is the frequency at which an elastic object naturally vibrates when hit, struck, or otherwise disturbed. Every system able to oscillate has its own natural frequencies. A pendulum, for example, always oscillates at the same frequency when set in motion. Its frequency depends only on physical properties such as the mass, length or stiffness of the spring. Furthermore, the amount of natural frequencies for a system depends on its degree of freedom and thus on its complexity. The lowest natural frequency of a system is called its fundamental frequency. If a forced vibration is applied to a system, at its natural frequency only a minimum of energy is required to keep it in vibration.

It is important to know the natural frequency of an object to predict its behaviour in relation to vibration. The most important reason for this is resonance. If a varying force with a frequency equal to the natural frequency is applied to a system, the oscillation will become violent. Its amplitude will increase highly and damages may occur. Although rare, total collapse is possible due to overloading or failure in fatigue (this scenario predominantly affects bridges).

The fundamental frequency for the simplest model of a dynamic system which has only one degree of freedom and no damping is given by:

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

In this case the natural frequency simply depends on the stiffness k and the total mass m of the system. This equation indicates the importance of these two qualities for a dynamic system. For concrete floors, the stiffness is composed of further three factors: it depends on Young's modulus, Poisson's ratio and the moment of inertia of the considered structure.

To minimise the perception of floor vibration it is important to achieve as high as possible values for the system's natural frequencies. This will occur if the stiffness is very high and the mass is by contrast very low. This is the ideal case which will result in very high frequencies. The opposite effect happens if a high mass in combination with a low stiffness starts to vibrate and therefore this situation should be avoided.

1.3 Cobiax flat slab

Cobiax is an international operating company which has developed a special solution in the lightweight flat slab system sector. Their product, the Cobiax flat slab, consists of hollow plastic spheres which are placed between the upper and lower static reinforcement of the slab (Figure 1.1, published by Cobiax Technologies AG). Each of these sphere are located in modules which consists of a steel cage including several balls. The cage avoids a contact between ball and static reinforcement which leads to an impairment of its bond. Additionally, a buoying upwards during concreting is avoided. The balls replace the concrete on its area with the lowest benefit. The main idea of this system is to remove the useless concrete which just produces dead load without improving the static qualities of the slab. The concrete

Figure 1.1 Cobiax cage modules

forms a hard shell with struts by using appropriately located cavities formed by hollow spheres. Nevertheless the slab has the same load bearing behaviour as traditional solid slabs and brings along some improvements to them. First of all Cobiax slabs weighs up to 35% less than solid slabs of equivalent dimensions. This has a positive effect on the number of necessary vertical bearing elements (up to 40% less column usage). It is also feasible to create large spans, up to more than 20 m, without using beams. These two factors increase the possibilities of open areas in buildings, making more alterations possible. Furthermore, the mass reduction is noticeable in designing foundations, leading to savings in the amount of material used. Other advantages include reductions in CO_2 emissions, savings in the amount of reinforcement needed, the application of all common standard designs and the smooth bottom view. This, the common formwork and the biaxial load bearing made possible by the hollow section's spherical shape are also advantages over common hollow concrete slabs such as waffle decks.

During the design process a few changes concerning the slab's specific qualities should be considered, including the decrease in stiffness for Cobiax slabs caused by the reduced moment of inertia compared to a solid slab. For this purpose numeric factors have already been determined and can be easily used for conversion (see Appendix C). Besides small modifications the whole design requires no other variation. At the building site the Cobiax system arrives in the form of cage modules for on-site use or as semi-precast slabs (Figure 1.2, published by Cobiax Technologies AG). Alternatively it can be used in combination with precast or composite slabs. The available ball diameters of the spheres range between 180 mm and 450 mm which allows for the production of slabs from 24cm to upwards of 60cm.

Figure 1.2 Cobiax semi-precast slabs

1.4 Aims and Objectives

The general aim of this dissertation is to investigate the impact of Cobiax flat slabs' specific qualities on their fundamental frequencies in comparison to solid concrete floors. As in both main numerical factors for the fundamental frequency the stiffness as well as the mass of the subject is decreased, the consequence for the slabs' behaviour in case of natural frequency should be researched. The result should clarify if and how these different qualities affect the slabs' performance. Furthermore it should deliver an overview of the treatment of concrete floors' natural frequencies in structural engineering.

This study investigates three objectives. Firstly, due to varying estimations concerning the limits of structures' natural frequencies, different evaluations following national codes and independent recommendations will be presented. Afterwards a compilation of simplified methods for hand calculations will be provided. They will be tested in some example calculations and their accuracy when compared with "finite element values" will be investigated. The third objective is the main area of this research. A precise comparison of Cobiax flat slabs and traditional solid slabs will be undertaken using finite element software. The data yielded will then be thoroughly analysed. This will result in a detailed evaluation of the characteristic qualities of a Cobiax flat slab.

1.5 Purpose of Research

This research is developed in collaboration with Cobiax Technologies GmbH, Darmstadt, Germany and Cobiax Technologies Ltd, London. To market their product effectively Cobiax must possess all necessary information concerning its structural behaviour. Some areas of performance are yet to be investigated, including the Cobiax slabs' behaviour in relation to natural frequency; a detailed comparison with a conventional concrete solid slab is also required. As already explained, natural frequency in this context depends on two major factors. The first is the stiffness of a slab which depends on its basic material as well as geometry. Compared to a solid slab the material property is the same but due to its hollow inside section, the Cobiax flat slab has a different geometry. This results in a lower stiffness, which indicates a decrease in its natural frequency.

The second key component of the equation for natural frequency is the mass. Contrary to the stiffness, where the hollow sections of the spheres have a negative influence, in the case of mass they are an advantage. The one-third reduction in concrete becomes perceivable and increases the value of natural frequency. The decrease of both values has an opposite effect; one will improve the solution and the other will impair it. The question is how much these characteristics impact on the product's performance and so which is to be the decisive factor in the final evaluation and recommendations.

The research will carry out a clear investigation of this problem and deliver an unambiguous judgement. It is important to explore all possible advantages of the Cobiax flat slab as, in the future, this research may help to convince clients worried about the effects of the slabs' lower stiffness on its dynamic qualities.

1.6 Literature review

1.6.1 Human Response to Floor Vibration

Arthur Bolton (1994) explained the importance of dynamics in structural engineering with the following sentence: “Crowds go to fairgrounds to be subjected to quite large accelerations and enjoy the sensation, but if subjected to a tiny fraction of that excitation in a building they might become sick or anxious.” The psychological effect of vibrating structures, then, can be profound. It can cause people discomfort, nausea or anxiety. Of course these are all extreme reactions, but milder effects present a difficult problem because of wide variations in human sensitivity. There is also dependence on whether subjects are alone or in a group; one particularly sensitive person amongst other people can sometimes trigger off a collective belief that a barely perceptible vibration is dangerous or uncomfortable. Another criterion of percipience vibration is the activity which is being pursued at the time. Usually people are more sensitive to vibration when they are in a quiet and untroubled area rather than, for example, a busy region. Furthermore the direction of vibrations affects the percipience. Investigations show that a translation in a horizontal direction has more effect than one in a vertical direction. As a result of all these factors it is difficult to set an accurate borderline between ‘acceptable’ and ‘unacceptable’ levels of vibration. It is only possible to provide data with ranges which are definitely unacceptable and thus should be prevented.

Reiher and Meister (1931) performed investigations to obtain such ranges. People of different ages, professions and provenances had to stand and lie on a vibrated platform. The tests covered sinusoidal vertical as well as sinusoidal horizontal vibration. The frequencies used started from 3 Hertz up to 70 Hertz and amplitudes from 0.0001 to 1.0 cm, figures which approach realistic values. During the tests, noise was an important factor. It was necessary to minimize noise as much as possible so as not to disturb hearing and thus the results.

After a vibration impact of 5 minutes every subject had to evaluate their “sentiences”. They had to organise their “sentiences” into six groups which ranged from “not perceptible” to “very disturbing”. After finishing all tests, the results were plotted in charts showing frequency against amplitude. All results were included in these charts and afterwards it was possible to draw borderlines for each category. These charts are one possible means of evaluating the consequences of vibration for the human body. In general Reiher and Meister recommend avoiding the last two categories. Residential areas should also avoid vibration from category 4 which translates as "keenly noticeable" vibration.

A similar investigation was carried out by Wiss and Parmelee (1974). They extended the amount of subjected persons from 10 to 40 and confined the scope of their tests to a standing position. All persons had to assess their perception using five classifications. For a steady-state condition (0 damping) this investigation showed a lower perceptible for a particular frequency and displacement compared of those performed by Reiher and Meister. However, the performance and analysis of the studies were not exactly the same and so could explain these differences. Further results concerned the effect of changing the damping for the perception of vibration. It was assumed that if the damping was increased from 0.02 to 0.20 of critical, the product of frequency and displacement would approximately double.

In general Bachmann (1987) recommends avoiding frequencies below 7.5Hz for office buildings made of reinforced concrete. This value ensures that even the third harmonic is taken into consideration, which means, that besides to the fundamental frequency its integer multiples are regarded. For example, if the frequency is f, the harmonics have frequency $2f$, $3f$, $4f$, etc. Compared to pedestrian structures such as gymnasia or sport halls where it is sufficient to regard the second harmonic, for office buildings it is necessary to consider the third harmonic as the occupants are more sensitive. Not only the occupants, however, should be protected against vibration. Human motions such as walking, running, dancing and skipping are sufficient to cause overstressing of the structure, and in extreme cases the loss of structural integrity, damage to non-structural elements (e.g. claddings), and development of cracking or excessive noise (e.g. due to reverberating equipment).

Brownjohn (2001) investigated the energy dissipation from vibrating slabs due to human-structure interaction. It was clarified how the presence of people located on a vibrating structure affects its dynamic behaviour. For this purpose a simply supported 7m x 1m x 0.075m prestressed concrete plank was forced to vibrate while a subject was standing on it. Five different sets of test were performed, including the subject sitting on a plastic chair, standing erect, with knees slightly bent, with knees very bent and finally with a solid mass equivalent to the subject.
The results confirmed that the human body acts dynamically with the structure by decreasing its natural frequency. This was explained by the fact that the effective mass is increased as well, but it was also identified that the human body has a beneficial effect concerning damping ratio because, depending on posture, damping can increase significantly.

1.6.2 Case Studies

The importance of both vibration problems and proper fundamental frequencies is shown by Hanagan (2005), who has published a paper containing several case studies on walking-induced floor vibration in existing buildings. This type of vibration is shown to affect different types of buildings including offices, a classroom and a clothing store. In each of these cases, the cause of vibration was people walking around the space.
In one case, occupants of an office building started to report annoying floor vibration in 2004. Interestingly, this building was constructed in 1974 and there had been no previous complaints concerning floor vibration. After an investigation measuring the acceleration of the affected slabs, it was detected that the fundamental frequency of this floor system was about 4.7 Hz. This value is not acceptable nowadays but it shows that even low frequencies might work well in special circumstances. After almost 30 years vibration problems occurred due to the removing of partitions which resulted in reduced damping and a higher vibration.
Another case study in this paper shows the importance of complying with current recommendations for natural frequencies. In this study structural engineers suggested a more substantial floor system, including a thicker slab, to meet

recommendations designed to avoid vibration problems. However, as a result of previous experience with this building type, the higher costs involved and the absence of vibration problems in the past, the developer elected for a thinner solution. Unfortunately he was wrong and the occupants perceived motions close to walk paths. The complaints stopped immediately after additional support and damping was created through the use of full-height partitions.

Further case studies were presented by Bachmann (1992). He published ten cases of vibration problems produced by human activity. One example specified serviceability problems in a two-story gymnasium. Every time the upper hall was used by fitness classes, floor vibration was noticed in the hall below and glazed exterior walls started to vibrate horizontally. Additional effects included rattling of doors and shutters and clattering of equipment. An investigation was carried out to determine the dynamical qualities of the floor. It was established that the fundamental frequency was about 4.9 Hz. When people jumped with a frequency of approximately 2.48 Hz, resonance was excited by the second harmonic. In order to avoid annoying effects and possible fatigue damage the fundamental frequency was improved to 7.3 Hz by increasing the floor's stiffness.
Other cases discussed in this paper showed similar problems caused by low fundamental frequency and excitation by humans which resulted in significant modifications being made.

1.6.3 Consideration of Vibration in Design

Fisher and West (2001) divided the consideration of human response to floor vibration into four major steps. First of all the natural frequency should be calculated which is affected by acceleration due to gravity, Young's modulus, moment of inertia, supported weight and the span of the structure. Afterwards the initial amplitude should be calculated. Damping of a floor is another essential factor as it affects the duration and nature of the vibration: physical tests showed damping percentages ranging from 3% for bare floors to 6% for finished floors and up to 13% for finished floors with partitions. The fourth factor is a standard of measure involving the previous three figures. For this purpose graphs are used, with axes showing the frequency and displacement amplitude. The human response to these factors is then plotted.

The adverse effects and therefore the necessary avoidance of resonance were illustrated in a report by Cooney and King (1988). It was claimed that, due to resonance, the motion of a floor may be magnified by up to 20 times its static load condition. A significant increase in acceleration, velocity and displacement occurs; an effect which should be avoided by all means. For this purpose the authors provided a design method to identify a possible risk of resonance for floors: specifically, vibration induced by human activities. First the expected load of the area including all participants and their activities had to be assessed. Occupants' activities will lead to an appropriate forcing frequency and their total load in combination with a particular factor will give the dynamic load. Special literature, for example the BS 6472 (1992), will provide values for the acceptable limiting of acceleration. The final steps are the determination of the total floor load including the dynamical load component and further the calculation of the fundamental frequency for the structure. With the help of these data and a special equation presented by the authors an initial check of potential resonance may be made. Where the acceptable level of acceleration was exceeded, increasing the stiffness was suggested along with relocating or controlling the activity or just accepting the discomfort.

The Canadian Institute for Scientific and Technical Information (1980) published a paper to provide a better understanding of vibration due to dynamical loads. The paper was specified to human induced vibration and gave a general overview of this topic. The maximum walking frequency for a person was given as 3 Hz; the approximate frequency for jumping was 5 Hz, but as also told, these values were unlikely to be reached. Also included were approximated equations for the fundamental frequency of simply supported, clamped and cantilever beams and uniaxial plates. Furthermore the relationships between static and dynamic deformation and vibration behaviour under periodic and single loads were shown by equations and examples. The example given of a group jumping in a gymnasium clarified that static deflection remains unchanged for different fundamental frequencies. However, the dynamical deflection increased for a reduced frequency and reached extreme values in the case of resonance.

Crist and Shaver (1976) complained about insufficient investigation of floor vibration in national codes. The accuracy of an evaluation for floor vibration can be complicated by such factors as a lack of necessary components. In addition the structure location, the type of structure, the type of occupancy and damping should be considered in literature. An additional cause for concern was the insufficient provision of data for evaluation which must then be qualified through further research.

Furthermore, this publication explained the complexity surrounding the determination of human activity and occupant response. Both are random variables. The dynamic load caused by the former depends on varying characteristic factors including walking gait, variation in weight, heel-to-ball of foot contact and footwear. Influences on the perception go beyond the technical values of frequency, direction and duration to encompass psychological factors in form of mental state, motivation and experience and the physical factors of sound and sight.

1.6.4 Two Way Hollow Decks

An overview of the general structural performance of biaxial hollow section slabs of the type Cobiax produces is presented by Pfeffer (2002), who investigated the slabs' bond between reinforcement and concrete, flexure load-bearing capacity, deflection and punching behaviour. Initial tests showed that due to the contact of spheres with reinforcement the bond between reinforcement and concrete decreased in these areas. This led to a development of reduction factors and furthermore to a suggestion for improving slab design by relocating the spheres from the reinforcement. However, the flexure load-bearing capacity of a two-axis hollow slab is comparable to a solid slab. If the concrete compression zone is above the sphere, it can be dimensioned as a rectangular cross-section by means of the usual methods. As a result of the decreased self-weight the bending performance of a hollow section slab is better than a solid slab. This occurs up to an external load-to-self weight ratio of 1:5. In case of punching it was discovered that load capacity is approximately 50% lower if spheres are included. To avoid this disadvantage, it is recommended that spheres are removed from inside the punching area. This results in a similar punching capacity to solid slabs and allows for a common punching design.

Another structural behaviour, the transverse force capacity, was investigated by Schnellenbach-Held (2003). A comparison between biaxial hollow section slabs and conventional concrete solid slabs showed the differences in their shear force performance. Though the load-bearing capacity before and after shear crack formation was similar, the failure load of the lightweight slabs was about 45% lower than the breaking load achieved with solid slabs. This can be explained by the reduced concrete area which decreases the transmission of tensile stresses. For slabs without shear reinforcement, this is the main impact on transverse force capacity.

1.6.5 Comparison of FEM and Field Tests

The results discussed in this research can be compared to the real behaviour of Cobiax slabs. Emad El-Dardiry et al. (2002) ran an investigation which yielded good results. He and his colleagues compared measured natural frequencies of an existing building with values calculated by different finite element models. For this purpose and as a part of the European Concrete Building Project (ECBP) a realistic office building was constructed inside the BRE Cardington Laboratory. It was a seven-storey in-situ concrete building consisting of long-span flat slabs supported by columns designed to Eurocode 2. Each floor was 3.75m high, giving a total height of 26.25 m. The building had three bays of 7.50 m constituting a width of 22.50 m and four bays of 7.50 m making a length of 30.00 m. All slabs were designed as reinforced concrete flat slabs with 0.25m thickness. The intended imposed load was 2.5 kN/m².

After finishing the construction, Building Research Establishment Ltd conducted dynamic tests on the floors. The tests involved monitoring the acceleration of the centre of each floor area in response to a heel-drop. The response was then converted to an autospectrum using a "Fast Fourier Transform" procedure and the dominant natural frequency was identified. All seven floors were covered by measurements from 11 different locations on each floor. The measurements provided a basis for evaluating the quality of different FE models. Consequently, FE analysis of several commonly used models was conducted, and the numerical and experimental results compared. The engineers used the FE software LUSAS and modelled different approaches to floor-column connection.

One result of this investigation was that, while the different models used in this study give different frequencies, the mode shapes are similar in a global sense. All approaches had a variance of between 2% and 17% from the measured values. The average difference was 12%. Another conclusion of a prior investigation was the negligible effect of mesh size on dynamic behaviour. In case of natural frequency all three meshes considered had no significant impact. However, they affected the appearance of the mode shapes and so a fine mesh was used.

A similar comparison was performed by Williams et al. (1993). Tests were carried out on reinforced and prestressed concrete floors of various configurations, covering the full range of spans and thicknesses encountered in typical structures. Newly cast, bare floors as well as already finished floors including false floors and services were tested. The building types tested included offices and car parks. These types are structurally quite similar with the exception of the lack of any finishes on the floor of the car park, which results in lower damping values.

The experimental set-up used a hammer test, in which a soft-tipped hammer generates the input excitation through a striking motion. By using other experimental equipment general vibration qualities such as natural frequencies, mode shapes or damping ration were determined. A single bay within the test floor was chosen as the test panel and divided into a 5 x 5 grid of equally spaced points. Afterwards every point was investigated five times to obtain an averaged response for each specific point. Later a finite element model was created using I-DEAS finite element software to compare the gained values.

A detailed comparison was given here for the specific example of a car park in Wycombe. The car park consisted of a 0.21m thick slab, supported by post-tensioned beams along column lines.

The results of this comparison show that the computer model gives very good estimates for the first three frequencies. All three frequencies are quite similar to those investigated. The averaged difference between both results is about 4%.
It was supposed that, due to the increasing importance of accurately representing the boundary condition, natural frequencies of higher modes would exhibit less similarity. However, as when assessing potential human discomfort due to vibration only the first few frequencies are important, the investigation concluded that it is possible to obtain a reasonable estimate of the dynamic characteristics of a floor by using finite element software.

Osborne and Ellis (1990) have presented a study of vibration design and testing of long-span lightweight floors, focussing on the estimation and evaluation of floor design. One major objective was the comparison between simplified hand calculations, computer supported calculations and accurate tests on-site. It was shown that all three, and especially the latter cases, predict similar values; the estimated frequency of a computer analysis was just 0.16 Hz (approximate 3%) higher than measured frequency.
Another interesting finding of this study was the change in dynamic behaviour from the bare floor to a finished floor including a false floor, service installations and fire protection. Although the finished floor showed only a small increase of damping and stiffness, qualitative observation by people performing a heel drop test agreed an improved perception.

The vibration assessment floor from Ove Arup & Partners (2004) provides particularly useful information because of its strong resemblance to the Cobiax flat slab system. The report includes the results of an investigation into the vibration behaviour of a floor for a typical hospital. For this case an idealised area of hospital floor was assumed. Its properties included 400mm thickness, 315mm ball size and 3 x 3 square bays. Each bay had a span of 9m x 9m. The imposed loads were estimated as realistic in-service values averaged over the entire floor area. Using the finite element software MSC NASTRAN, a model was created to analyse the floor's dynamic performance. The slab was modelled as a 400mm thick solid slab and its specific qualities were considered by a reduced stiffness and mass. The analysis showed that the fundamental frequency of this floor is 11.8Hz. Furthermore a footfall response analysis was carried out to obtain the root-mean-square (r.m.s.) velocity of the floor. Afterwards all results were compared with a floor of 400mm solid concrete. The first natural frequency reduces to 10.4Hz, a decrease of 12%. The responses for Bubbledeck slabs are 16% higher than those for a solid slab of the same 400mm thickness.

1.6.6 Determination of Frequency

Mazumdar (1971) determined the fundamental frequency of elastic plates of arbitrary shape by aid of constant deflection lines. For this purpose he assumed the classical small-deflection theory to be valid. His method for the case of elliptical plates was illustrated specifically because of its increased complexity compared to other shapes. The assumption that the lines of equal deflection also had an elliptical shape was made in response to the problem of determining the resulting time-dependent deflection field. This approximation is then only valid for slender elliptical plates, making this method only practical for thin plates. After the determination of all necessary dynamical equations, two examples were calculated. One plate was supposed to have clamped edges and the second was simply supported, an estimation which had previously only been published in one work. Furthermore the author compared his method with results already established in literature. For small ratios of both semi-lengths this comparison showed very similar outcomes to the other present values.

Jones (1975) used this method and extended the comparison. He investigated simplified calculations for the fundamental frequency of structures with different shapes and boundary conditions such as equilateral triangular, rectangular or semicircular plates. Afterwards he also compared these approximations with computed and more exact values. As before, the results of this comparison were very good. For the example of a clamped quadratic plate, the difference between the two estimations was 0.05%.

Magrab (1976) adopted a different approach to estimating the natural frequencies for plates. He derived an expression for orthotropic rectangular plates with simply supported, elastically supported or clamped boundary conditions. Instead of using existing estimation methods and thin-plate theory which relies on a length-to-thickness ratio he solved the problem with another mathematical technique: the Mindlin-Timoshenko theory. This theory is an improvement on the Euler-Bernoulli beam theory, which condensed a beam to a 1-D structure. Another assumption of this theory is that the plane cross-section of a beam remains plane and normal to the reference line when the beam deforms due to bending.
In addition to this hypothesis Timoshenko's theory considers sheer and rotational inertia effects and the resulting deformation. Comparing an example with other estimations which use the thin-plate theory yielded analogical values with differences between 0.08% and 3.7%. Excepting the estimate values of Elishakoff (1974), all other fundamental frequencies are higher than those calculated by the author. This results from the consideration of transverse sheer and rotary inertia which also imbibes vibration energy additional to bending as required by the thin-plate theory.

The influence of Timoshenko's additional consideration, rotational inertia and sheer deformation for rectangular plates, was formerly investigated by Mindlin, Schacknow and Deresiewicz (1956) who determined a method to obtain natural frequencies with coupled modes. Special regard was given for the case of a plate with one pair of parallel free edges and the other pair simply supported.

Leissa (1973) presented a study of approximate formulas for free vibration of rectangular plates. It was the first compilation of all 21 cases which involved all possible combination of classical boundary conditions, like clamped, simply supported and free edges. Amongst other techniques he used the Ritz method or the beam function for this purpose. This led to the production of a set of 21 tables for the estimation of the first 9 modes for each plate including different length-to-width ratios. Furthermore the effect of changing Poisson's ratio on the natural frequencies was presented. In every case the frequency depends on Poisson's ratio. An example of a plate supported on two parallel edges by simply-supports and on the other a pair of free edges showed that increasing Poisson's ratio caused a decrease in natural frequency. Other objectives of this investigation were the evaluation of accuracy compared to the referenced Warburton's formulas for natural frequencies and the effect of changing edge condition upon the frequencies and their accuracy.

2. Human Response and Acceptance Criteria

Evaluation of measured or calculated values of floor vibration must be carried out in order to predict its influence on the surrounding environment. This requirement creates the need for specific acceptance criteria. It is possible to classify the effects floor vibration has on its environment into three main areas:

- Overstressing of structural members
- Physiological effect on people
- Impact of production processes with sensitive equipment or susceptible machinery in general

(Bachmann and Ammann, 1987)

Of these three, most attention is paid to human response. This is because damage and fatigue failure of structural elements due to walking-induced floor vibration are unusual, and every different type of machinery has its own very specific requirements. Different acceptance criteria and recommendations have been developed to measure human response. Unfortunately, though, it is not possible to provide exact limit values and this can obviate perception of motion. As the variety of human responses to floor vibration varies greatly, these criteria can only utilise reference values gained by experience or field tests. The complexity of both perception levels and human sensitivity to vibration is illustrated by a high number of interrelated factors. Among them are:

Direction of motion: Humans evaluate every direction of motion differently. Generally vertical foot-to-head vibration is considered more annoying than horizontal chestto-back vibration (Cooney and King 1988). However, every direction of motion has to be considered because of its potential occurrence. While horizontal vibration causes only small concern in offices and other workplaces, its importance increases in the design of residences and hotels where sleeping comfort must be considered.

Personal characteristics: Different responses are given depending on the age, sex and level of concentration of the subjects as well as those of surrounding community.

Timing and duration: Motions at night are less tolerated than those occurring during the day. Furthermore continuous motion (steady-state) is more annoying than motion caused by infrequent impact (transient).

Expectation: If subjects are forewarned of vibration, their perception will be less sensitive

Current activity: Different levels of acceptance exist for office work, physical work, resting, dining and dancing. Acceptance levels are also affected by the surrounding environment (e.g. home, office or gymnasium).

Since the pioneering work of Reiher and Meister (1931), most vibration criteria provide graphs defining regions of acceptable and unacceptable vibration. Usually these are plotted in frequency versus peak acceleration due to gravity of the floor vibration, but other numerous parameters such as velocity or displacement of the treated floor can be included. On the graph, single lines represent a constant level of human reaction (isoperceptibility lines) with the region above a line denoting unacceptable vibration. These act as boundaries between different levels of perception.

2.1 Recommendations in Codes

2.1.1 British Standard

In British Standard BS6399-1:1996 Annex A, two different approaches are recommended for the design of domestic and residential structures, especially single family buildings. In areas subjected to dancing or jumping there can be an increased risk of unpleasant floor movement and even resonance may occur. In order to avoid this phenomenon, it is recommended that vertical natural frequency is limited to at least to 8.4Hz and horizontal natural frequency to a minimum of 4.0 Hz. These frequencies should be calculated for the empty structure.
Another approach is to consider dynamic loads as well as dead and static imposed loadings during the design stage. Deformation due to dynamic loads should not exceed limits appropriate to the building or structure type.

No detailed specifications are provided for lightweight and long span structures. Only the general advice of taking floor vibration into account and the recommendation of specialist guidance documents are given:

> Where lightweight and long span structures are used as concourses and public spaces, they are likely to be subjected to inadvertent or deliberate synchronized movement by people, causing dynamic excitation. The design provisions should take account of the nature and intended use of the structure, the potential number of people and their possible behaviour. Structural design should be undertaken with the help of specialist advice and specialist guidance documents, as required by the appropriate certifying authority.

A more detailed treatment of floor vibration is covered by the British Standard BS 6472:1992 “Guide to evaluation of human exposure to vibration in buildings (1 Hz to 80 Hz)”. This guide has a general approach, for application to many vibratory environments. It is applicable to vibrations transmitted through the supporting surface to the body as a whole by considering different positions and all three axes, as defined in figure 2.1.
Within the range of 1 to 80 Hz, the guide also considers different types of structures including offices, residential buildings or critical working areas such

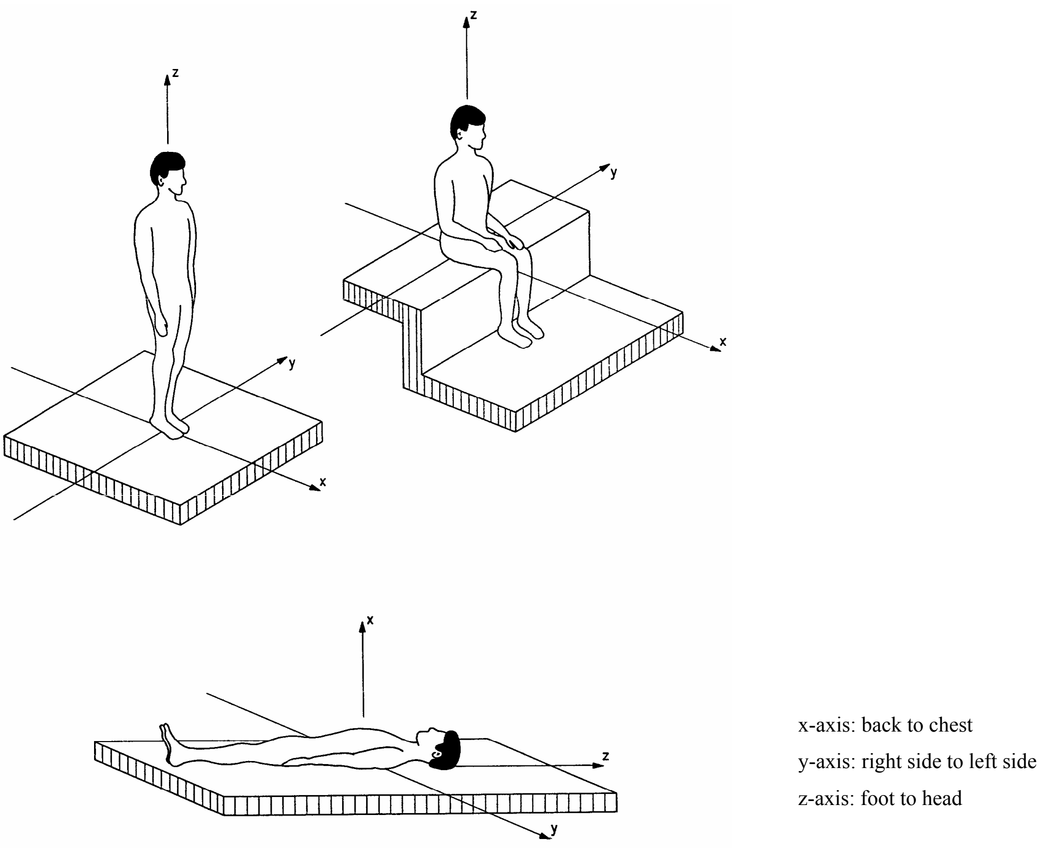


Figure 2.1 BS 6472 (1992) Coordinate systems for vibration influencing humans

as operating theatres. All allowable vibrations are provided in curves of annoyance for humans in terms of direction of transmission, frequency and acceleration or velocity. While acceleration is given as r.m.s. acceleration (root-mean-square acceleration), velocity is specified as a peak value. In terms of human response the British Standard divides vibrations into two classes: impulsive and continuous vibration.

Impulsive vibration is defined as a rapid build-up to and decrease from a peak; for example vibration caused by the impact of a single heavy object on a floor. This type may also consist of several cycles of vibration providing that duration is short (less than approximately 2 seconds). The other category describes continuous vibration which remains uninterrupted over a certain time period (for example vibration caused by a group of people walking). Their different consideration is given by individual multiplication factors shown in table 2.1. These factors are used to multiply the base curves and obtain the according curve for a specific case.

Place	Time	Multiplying factors (see notes 1 and 5)	
		Exposure to continuous vibration [16 h day, 8 h night] (see note 2 and Appendix B)	Impulsive vibration excitation with up to 3 occurrences (see note 8)
Critical working areas (e.g. hospital operating theatres, precision laboratories (see notes 3 and 10)	Day	1	1
	Night	1	1
Residential	Day	2 to 4 (see note 4)	60 to 90 (see notes 4 and 9, and Appendix B)
	Night	1.4	20
Office Day	Day	4	128 (see note 6)
	Night	4	128
Workshops	Day	8 (see note 7)	128 (see notes 6 and 7)
	Night	8	128

NOTE 1 Table 5 leads to magnitudes of vibration below which the probability of adverse comments is low (any acoustical noise caused by structural vibration is not considered).

NOTE 2 Doubling of the suggested vibration magnitudes may result in adverse comment and this may increase significantly if the magnitudes are quadrupled (where available, dose/response curves may be consulted).

NOTE 3 Magnitudes of vibration in hospital operating theatres and critical working places pertain to periods of time when operations are in progress or critical work is being performed. At other times magnitudes as high as those for residences are satisfactory provided there is due agreement and warning.

NOTE 4 Within residential areas people exhibit wide variations of vibration tolerance. Specific values are dependent upon social and cultural factors, psychological attitudes and expected degree of intrusion.

NOTE 5 Vibration is to be measured at the point of entry to the entry to the subject. Where this is not possible then it is essential that transfer functions be evaluated.

NOTE 6 The magnitudes for vibration in offices and workshop areas should not be increased without considering the possibility of significant disruption of working activity.

NOTE 7 Vibration acting on operators of certain processes such as drop forges or crushers, which vibrate working places, may be in a separate category from the workshop areas considered in Table 3. The vibration magnitudes specified in relevant standards would then apply to the operators of the exciting processes.

NOTE 8 Appendix C contains guidance on assessment of human response to vibration induced by blasting.

NOTE 9 When short term works such as piling, demolition and construction give rise to impulsive vibrations it should be borne in mind that undue restriction on vibration levels can significantly prolong these operations and result in greater annoyance. In certain circumstances higher magnitudes can be used.

NOTE 10 In cases where sensitive equipment or delicate tasks impose more stringent criteria than human comfort, the corresponding more stringent values should be applied. Stipulation of such criteria is outside the scope of this standard.

Table 2.1 BS 6472 (1992) Multiplying factors

Figure 2.2 shows one example of a multiplied curve where the frequency is plotted against the r.m.s. acceleration. It is recommended that the frequency-acceleration combination is kept below the line which corresponds to the relevant case, therefore minimising adverse comments or complaints of vibration.

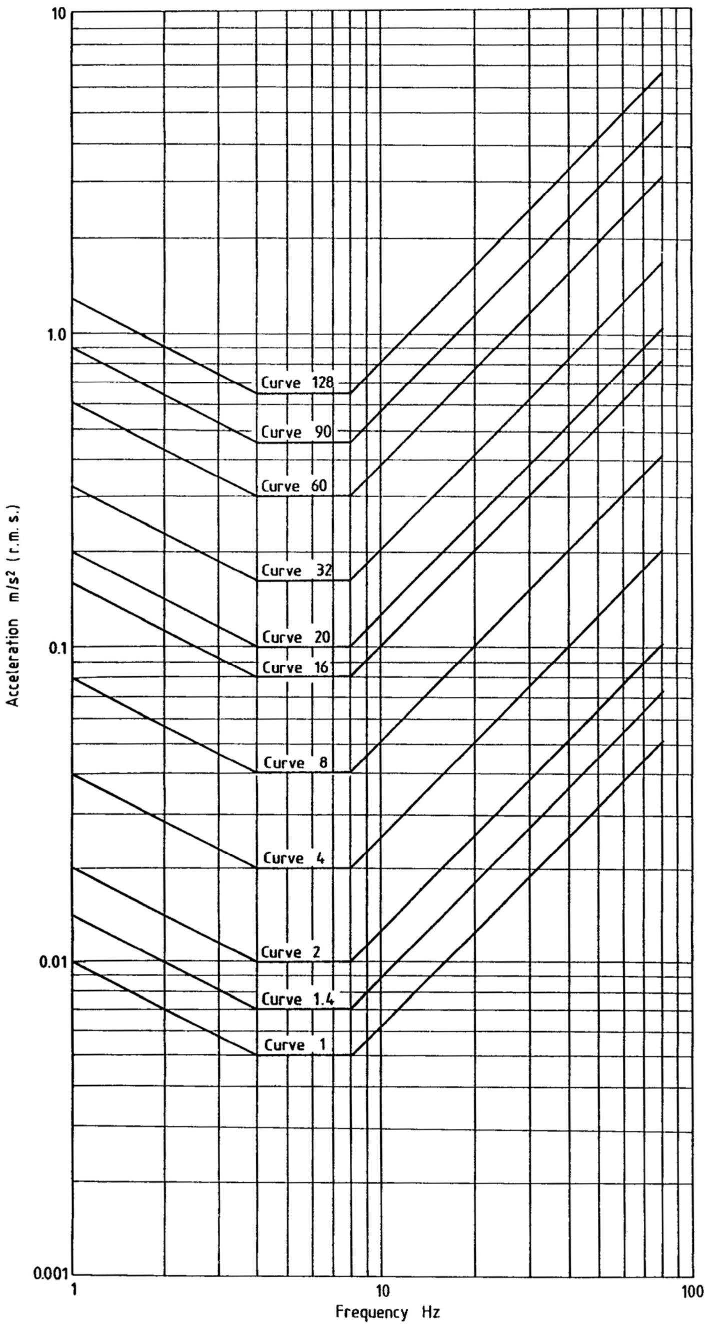


Figure 2.2 BS 6472 (1992) Building vibration z-axis curves for acceleration (r.m.s.)

Another method for specifying satisfactory vibration magnitudes is provided in Appendices A and B. By calculating and comparing the vibration dose value with limit values presented in tables it is also possible to evaluate a structure's vibration; however, this approach is exclusively provided for residential buildings and is therefore only applicable to a small amount of problem areas.

2.1.2 German Standard

The German Institute for Standardisation published a similar code entitled DIN 4150-2 (1999) "Erschütterungen im Bauwesen; Einwirkungen auf den Menschen in Gebäuden". This code provides recommendations concerning humans' vibration perception in residential and similarly used buildings and is applicable to periodic as well as non-periodic vibrations. It also deals with frequencies from 1 to 80 Hz and considers all three axes of a human body as well as different types of buildings; but in contrast to the English code, the German DIN uses a modified parameter called *KB* value, which depends on the frequency of motion and was established to assess the acceptance of motion by limiting the frequency to specified values (see table 2.2).
As in the BS6399-1(1996), the limit criteria in the DIN 4150-2 (1999) depends on occupancy and time of day.

Place	Day			Night		
	A_u	A_o	A_r	A_u	A_o	A_r
Areas with commercial buildings and as an exception residencies for occupants or directors of these companies	0.4	6	0.2	0.3	0.6	0.15
Areas with commercial buildings predominantly	0.3	6	0.15	0.2	0.4	0.1
Areas without predominance of commercial buildings as well as residences	0.2	5	0.1	0.15	0.3	0.07
Areas with residential buildings predominantly	0.15	3	0.07	0.1	0.2	0.05
Critical areas (e.g. hospitals)	0.1	3	0.05	0.1	0.15	0.05

Table 2.2 Extract of DIN 4150-2 (1999); reference values *A* for residential and similarly used buildings

For the comparison of measured and recommended limit values two different *KB* parameters are used. These are represented by KB_{Fmax} for the maximum motion and KB_{FTr} , which is an averaged value spread over the assessment time. These two values must be estimated for each of the three axes in which motion may occur. The worst case becomes decisive.
Once both critical values are known, a fixed procedure shown in figure 2.3 can be applied. KB_{Fmax} and in special cases KB_{FTr} need only be compared to reference values A_u and A_o and a final evaluation is then given.

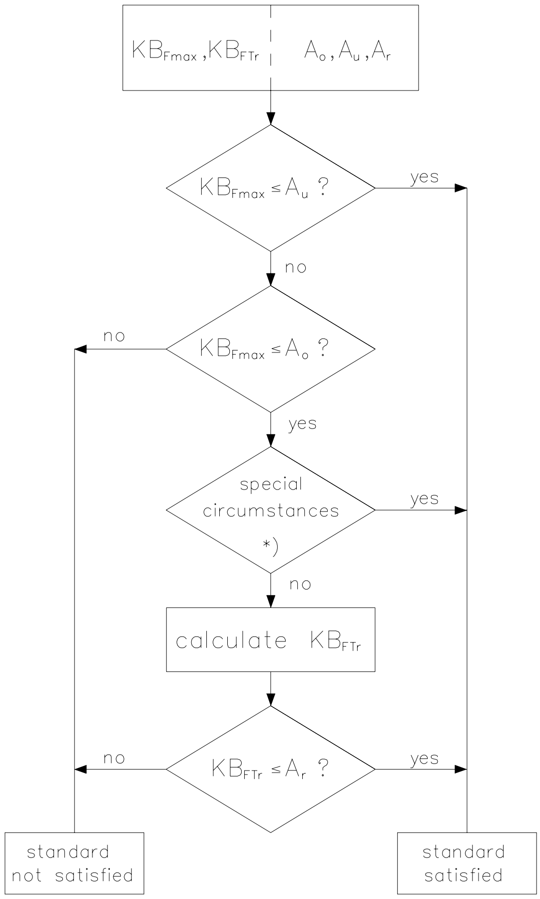


Figure 2.3 DIN 4150-2 (1999) progression of assessment procedure

This method of predicting vibration acceptance is more complex than that of the British code. The calculation of all necessary values requires a lot of time and precise knowledge of the circumstances, such as duration of impact. It seems to be a method for evaluating measured values rather than calculated values from the design stage.

2.2 Recommendations in Literature

Besides national codes and standards, many independent and individual recommendations are available. These are partly developed from practical tests on subjects and partly gained by experience of existing buildings. The following paragraphs will deliver an overview of some of the recommendations found in literature.

The most frequently cited reference in the field of human acceptance and floor vibration is Reiher and Meister (1931). These authors carried out the first research on this topic by investigating how horizontal as well as vertical vibration affects humans. Subjects were placed on shaking tables which varied in amplitude and frequency. Afterwards, they had to rate the motion using one of six categories. This information then made it possible to plot the relationship between amplitude and frequency in relation to human perception. It should be mentioned here, though, that due to the long duration (approximate 5min.) of each test the results should be applied to continuous rather than impulsive vibration. The Reiher–Meister scale for vertical vibration is shown in figure 2.4.

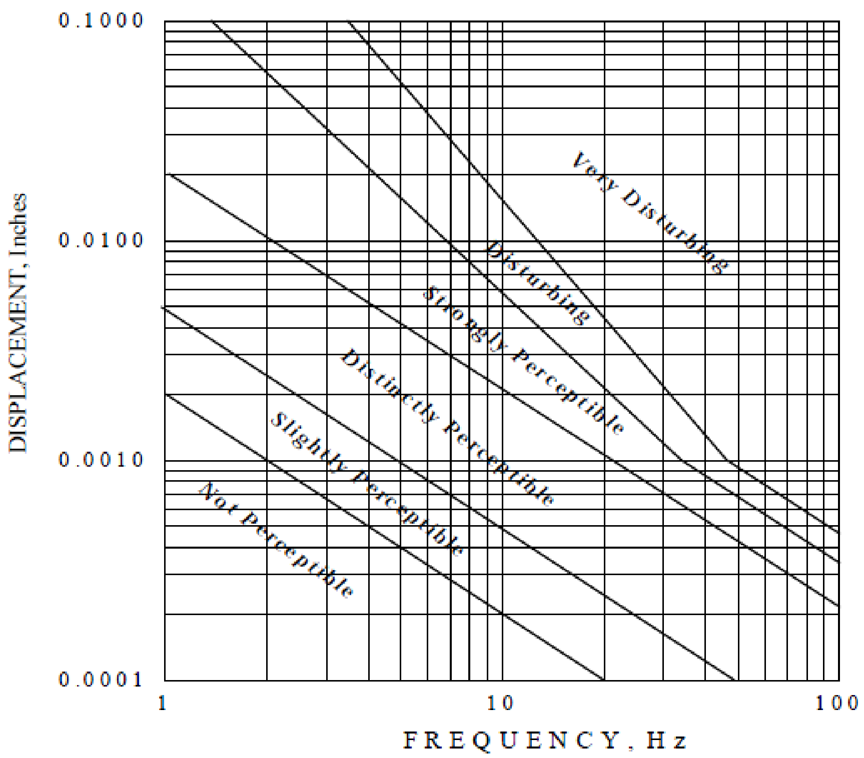


Figure 2.4 Reiher-Meister scale

Lenzen (1966) determined that damping and mass, and not stiffness, were the most important parameters in preventing unacceptable floor vibration caused by walking. He suggested that if vibration is reduced by damping to a negligible quantity in 5 cycles the human will not respond, whereas if it persists beyond 12 cycles a steady-state vibration is noticeable.

He also modified the Reiher-Meister scale by increasing displacement by a factor of 10 (Figure 2.5). The difference results from discriminative human sensitivity against the duration of vibration. In contrast to Reiher and Meister's steady-state vibration, Lenzen developed this criterion for transit vibrations which have a reduced effect on subjects.

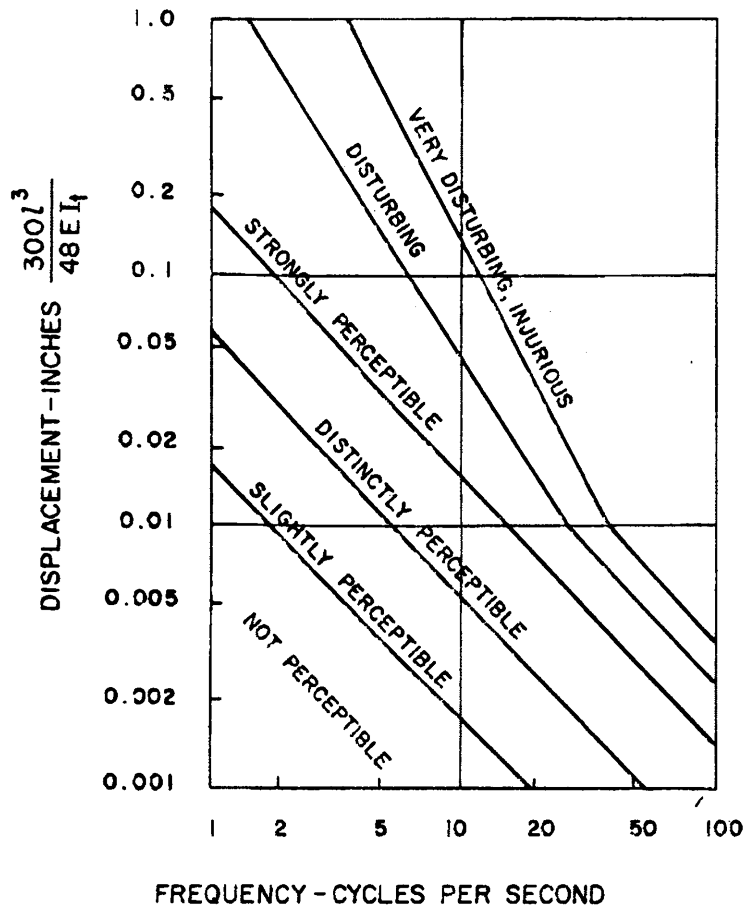


Figure 2.5 Graph of reduced human response

Allen and Rainer (1976) developed annoyance criteria for walking vibrations in terms of acceleration and damping based on tests using 42 long-span floor systems. These were then incorporated into the Canadian Standards Association's national code. The proposed criterion (figure 2.6) is an extension of Lenzen's work and considers continuous vibration (10 to 30 cycles) as well as walking vibration. They are suggested for use with quiet human occupancies, for example residences, offices or schoolrooms. Pernica and Allen (1982) modified the criteria for active occupancies such as shopping centres and car parks by increasing the limits by a factor of 3.

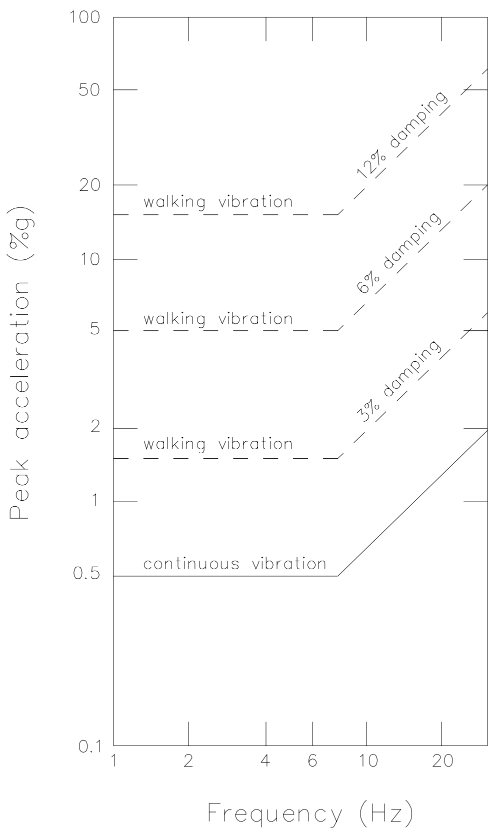


Figure 2.6 Annoyance criteria by Allen and Rainer

Interpretation of this graph requires care, because both types of line are a criterion for floor vibration. The continuous vibrations are caused by a person walking on a floor, as are the walking vibrations. The difference between these types is that instead of the continuous vibration line which uses average peak acceleration to assess acceptability, the walking vibration lines represents the initial peak acceleration resulting from a heel drop test.

Allen and Murray (1993) mentioned the necessity of the first three harmonics in avoiding resonance. The third harmonic should be considered for a single person walking with normal velocity. For jogging or more than one person, only the first two harmonics are important. If the number of persons walking on a structure increases then the dynamic loading does increase, but at the same time lack of coherence at higher harmonics increases. However, generally such cases are rare enough to not be a problem in practice.

The proposed design criterion for the acceptance of floor vibration is provided through different approaches. One of them can be expressed in terms of fundamental frequency and is given by:

$$f_0 \geq 2.86 \ln\left[\frac{\mathrm{K}}{\beta \mathrm{W}}\right]$$

Where f_0 is the fundamental frequency, W is the weight and K is a constant given in table 2.3 which also provides approximate values for the damping ratio β.

	K kN	β
Offices, residences, churches	58	0.03*
Shopping Malls	20	0.02
Footbridges	8	0.01
*0.05 for full-height partitions, 0.02 for floors with few non-structural components (ceilings, ducts, partitions, etc.) as can occur in churches		

Table 2.3 Values of *K* and *β*

Bolton (1994) emphasised the importance of acceleration for the perception of floor vibration by using the example of passengers in an aircraft. As the craft flies with high and constant speed, the humans inside do not feel any movement. It is the change of velocity, that is, the acceleration, which is perceived. The acceleration is directly proportional to the square of the frequency and to its amplitude of displacement.

$$\text{Acceleration} = -(2\pi f)^2 \times \delta$$

Provided with knowledge of the fundamental frequency and acceleration due to gravity, it is possible to evaluate the effect of floor vibration on humans. To this end, Bolton proposed to divide the range of frequencies into two areas: from 0 to 10 Hz and above 10 Hz. The relationship between acceleration and human perception is presented in table 2.4 below:

	acceleration [m/s²]	
	$f_0 \leq$ 10 Hz	$f_0 >$ 10 Hz
barely perceptible	0.03	0.0005
clearly felt	0.10	0.0013
unpleasent	0.50	0.0067
entirely unacceptable	2.00	0.0133

Table 2.4 Human perception criteria by Bolton

A more general recommendation was proposed by Bachmann and Annmann (1987). They argued that due to the lack of exact knowledge concerning various floor parameters and their inter-relations, it would be more practical to provide rough limit values for the designer's use. Their recommended values for natural frequency as well as the acceleration of floor vibration are shown in table 2.5.
The fact that the frequencies increase for different construction materials is due to their decrease in stiffness, mass and damping.

	f_0 [Hz]				acceleration [m/s²]
	reinforced concrete	prestressed concrete	composite	steel	
Offices	> 7.5	> 8.0	> 8.5	> 9.0	≤ 0.5 - 1.0
Gymnasia and sport halls	> 7.5	> 8.0	> 8.5	> 9.0	≤ 0.2
Dancing and concert halls	> 6.5	> 7.0	> 7.5	> 8.0	≤ 0.5 - 1.0

Table 2.5 Overall acceptance levels for various types of environment

Similarly to Allen and Murray (1993), Bachmann and Annmann also mentioned observing more than frequency; while 'more active' areas like sport halls, dancing or concert halls should be considered using the second harmonic, they propose high tuning with respect to the third harmonic of load-time function for offices or other quiet working places.

Other literature presents this topic with less accuracy and provides only rough limit values for consideration during the design of floor systems.
For the prevention of resonance Fisher and West (2001) recommend avoiding natural frequencies of between 1 and 4 Hz for ‘walking areas’ and 5 Hz for ‘dancing areas’.
Cooney and King (1988) mentioned that crowds involved in activities such as dancing or gymnastic can be synchronised by music or other means up to frequencies of 6 Hz. Beyond this limit they become uncoordinated and a random forcing function results. Therefore they suggest checking floors with natural frequency below 6 Hz and possible support of assembly occupancies for resonance.
Furthermore, Hanes (1970) reported that studies using automobile and aircraft passengers showed that the natural frequency of human internal organs is between 5-8 Hz. Therefore, floor systems with natural frequencies in that range could possibly cause human discomfort and should be avoided.
Morrison (2006) specified the interfering frequencies of individual sub systems within the body. Some examples are the abdomen-thorax region, 3 Hz, the spine, 5 Hz or the heart, 7 Hz. The frequencies at which the whole human body is most sensitive are 3 - 6 Hz and 10 - 14 Hz.

2.3 Summary

The effects of vibration on humans vary so widely that evaluation is very complex and depends on many factors. In general, it is difficult to set accurate limits to any parameters. However, past research has attempted to obtain boundaries for different kinds of structures and activities. These should result in improved ambience in new structures as well as decreasing the chance of justifiable complaints made by users.

3. Simplified Hand-Calculation Methods

In the past, there have been many attempts to find simplified methods for calculating the fundamental frequency of structures. Before finite element software and computer packages, which predict all necessary information accurately within a few seconds, this information needed to be estimated. Even today, when computers are used universally, additional hand calculations are still recommended because they help to give an initial assessment and are able to forecast critical areas before or during the design stage. Alternatively, they might be used as an additional check on results calculated by a computer (Weber 2002).
Simple equations and tables make it possible to estimate natural frequencies in a very short space of time. Furthermore, only a few predictions are necessary, these being material properties such as the Young's modulus or the Poisson ratio and geometrical properties such as thickness of the structure or the length of span. Equipped with this data and a simple calculator, fundamental frequency can be predicted in seconds, provided the structure is not too complex. As the frequencies of complex structures would require huge efforts to hand-calculate, methods are only published for simple models such as one-span or continuous beams, one-span slabs or chimneys and pylons. The simplification is usually based on beam theory and can easily be adopted for these kinds of structures.
Another simplification of these methods is the assumption of boundary condition.
In reality, the grade of restraint for each support is unclear. Obviously in practice the support conditions are rarely simply supported or truly fixed, but in simplifications all supports are assumed to be 100% simply supported or clamped. This produces only small inaccuracies compared with the real dynamic behaviour of structures.
Other factors can also affect the accuracy between model and real behaviour. There are some influences which a simplification cannot or will not predict. An example of one such influence is the dissipation of energy due to contributions from coating or a suspended ceiling. This increases the damping ratio and therefore counteracts the vibration. Brownjohn (2001) also highlighted that a certain amount of vibration is reduced by occupants standing on the floor, a factor which cannot be predicted in hand calculations. Another difficulty is the non-linearity which may occur after cracking. Once the concrete starts to crack, the

effective stiffness is in flux. Due to the smaller amount of concrete cross-section, the moment of inertia decreases, resulting in a higher natural frequency.
These are a few circumstances which prevent 100% accurate values from being made; nevertheless, in many cases simplified estimations can provide good practical results and therefore initial ideas concerning the dynamical behaviour of the structure. Thus, problematical areas can be identified in an early design stage.

This chapter deals with simplified methods for calculating the fundamental frequency of slabs. For this purpose a summary of some existing literature providing equations or tables is presented and is checked against their accuracy. Therefore, calculations for a set of common structural slabs considering free undamped natural frequency for rectangular isotropic plates are carried out (Appendix A). Afterwards each example is compared against more exact values calculated by finite element software, enabling a general evaluation of their practical use to be undertaken.

3.1 Common Mathematic Techniques

When estimating the necessary values of eigenvalues, natural frequency or mode shapes, simplified methods usually involve different mathematical techniques. The main ideas of two different approaches, the Stodola method and the Rayleigh method, are described briefly below.

Referring to Caverson., Waldron and Williams (1994) the best-known approach is the Stodola method. This is an iterative method, in which the shape mode of an element is estimated and the initial forces associated with this mode shape are determined. A static analysis is carried out concerning these forces, providing a deflected shape for the condition. Instead of the previous estimated mode shape, the new deflection shape is now used for the same procedure. These iterations are then repeated until a sufficiently accurate solution is achieved.

The Rayleigh and Rayleigh-Ritz methods are also widely used to predict the natural frequency of structures. The Rayleigh method is based on the principle of energy conservation: if an undamped freely vibrating spring-mass system is assumed, no absorption of energy will take place and so the amount will remain constant. The relationship between the involved energies, potential and kinetic, requires their maximum values to be equal. The equations for both energies can then be computed and an equation for the fundamental frequency of a one-mass-system formed. This principle is then used when analysing systems with a greater degree of freedom. In this case, the mode shape of the fundamental frequency needs to be assumed first. Further on, it should be noted that, while the amplitude of motion varies with time, the shape of vibration does not, leading to an expression by a shape function. The assumed shape function simplifies the structure to a single degree of freedom system. The initial method of equating the energies can then be applied and the fundamental frequency calculated (Clough and Penzien 1993). The Rayleigh-Ritz method is an extension of Rayleigh's method and determines the natural frequency in the second or higher order. It uses the basic concept of Rayleigh's method and minimises the total energy to its original amount after adding the assumed shape function to gain the lowest frequency. This method can be applied iteratively as well as partially. The most important step within the method is to make a realistic assumption of the mode shapes in order to avoid a high number of iterations.

3.2 Formulas and Tables for the Calculation of Fundamental Frequency

3.2.1 Equivalent Beam Method

A widely used technique for determining the fundamental frequency of a floor is the equivalent beam method (Lenzen 1966). However, the assessment of beams is only recommended for the estimation of one-way spanning systems and not for two-way spanning floors. For this reason it is more often used for composite floors than concrete slabs.

Referring to Caverson, Waldron and Williams (1994), this equation should provide an accurately estimated frequency for composite as well as one-way spanning concrete floors. The first natural frequency is determined by:

$$f_0 = \frac{\pi}{2 \cdot a^2} \sqrt{\frac{E \cdot I}{m}} \tag{1}$$

3.2.2 Equivalent Plate Approach

In addition to the equivalent beam method only applicable to one-way spanning concrete slabs, Williams and Waldron (1994) as well as Jeary (1997) presented an equation for the fundamental frequency of simply supported two-way spanning slabs. Thanks to the influence of all three dimensions and the flexure rigidity of the plate *D*, it incorporates biaxial slab properties and provides good solutions.

$$f_0 = \frac{\pi}{2}\sqrt{\frac{D}{m}}\left(\frac{1}{l_x^2} + \frac{1}{l_y^2}\right) \tag{2}$$

3.2.3 Concrete Society Method

Another calculation procedure for biaxial slabs was proposed by the Concrete Society (2005). As with the equivalent plate approach, it uses an approximation of the equivalent beam method and considers the increased stiffness of a two-way spanning floor. This leads to two independent orthogonal modes occurring for both directions individually. The lower of both natural frequencies can be considered as the fundamental frequency for this slab. A general advantage of this method is its multifarious application to different types of slabs including solid, ribbed and waffle, and its additional consideration of several bays for each direction. However, the following equations are just presented for solid slabs in one direction (x-direction). The characteristics of the second direction mode are determined by interchanging the x- and y-subscripts in these equations.
The first step is to define the effective aspect ration of the slab by:

$$\lambda_x = \frac{n_x\, l_x}{l_y}\left(\frac{EI_y}{EI_x}\right)^{1/4} \tag{3}$$

Afterwards it is necessary to calculate the modification factor k_x:

$$k_x = 1 + \frac{1}{\lambda_x^2} \tag{4}$$

The natural frequency for slabs with perimeter supports is:

$$f_x^{'} = k_x \frac{\pi}{2}\sqrt{\frac{EI_y}{ml_y^4}} \tag{5}$$

For slabs without perimeter supports, the latter equation has to be modified by calculation of an additional frequency f_b:

$$f_b = \frac{\frac{\pi}{2}\sqrt{\frac{EI_x}{ml_x^4}}}{\sqrt{1+\frac{EI_x l_y^4}{EI_y l_x^4}}} \tag{6}$$

The final natural frequency can now be obtained by:

$$f_x = f_x^{'} - \left(f_x^{'} - f_b\right)\left(\frac{\frac{1}{n_x}+\frac{1}{n_y}}{2}\right) \tag{7}$$

3.2.4 Static Deflection Method

As seen in paragraph 3.1, it is possible to calculate the natural frequency by aid of the kinetic and potential energy within the structure. As a result of the associations of both, kinetic energy with the motion of mass and potential energy with the strain energy stored in the elastic structure during deformation, a relationship between the natural frequency and the deflection of a structure exists. This relationship can be used to calculate the fundamental frequency for structures whose static deflection is known. The derivation of an equation for an example of the spring-mass-system demonstrates the relationship well. The static deflection of a mass attached on a spring is given by:

$$\delta_S = \frac{m \cdot g}{k} \tag{8}$$

The natural frequency is already known as:

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \tag{9}$$

Now it is possible to incorporate equation 3.4.1 into equation 3.1 which gives:

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{\delta_S}} \tag{10}$$

The final equation shows that only the acceleration due to gravity and the maximum static deflection are required to obtain the fundamental frequency. But due to an additional factor, the length of span is involved now, this method should be more accurate than that of equation 1. Of course, this depends on the accuracy of the estimated static deflection but the higher amount of necessary data used in the calculation of static deflection allows this initial conclusion.

A modified version of this method was published by Blenvis (1979), who used the expression derived by Mazumdar (1971) and modified by Jones (1975) for calculating fundamental frequency. These earlier authors developed a method to estimate natural frequency aided by the constant deflection lines of an element. Although this equation was developed for clamped elliptic plates, it also predicts the first frequency of plates of various shapes and boundary conditions. The new equation is now:

$$f = \frac{1.277}{2\pi}\sqrt{\frac{g}{\delta_S}} \tag{11}$$

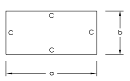
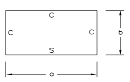
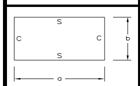
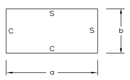
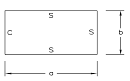
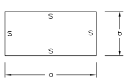
3.2.5 Approximation Presented by Hearmon

Hearmon (1959) presented approximations for the estimation of rectangular orthotropic plates using the already derived expressions from Warburton (1954) and extending them to orthotropic plates with a combination of clamped or supported edges. These approximations are based on the Rayleigh method and assume that the nodal lines of the deflection are approximately parallel to the sides of the slab. Furthermore, it is supposed that all three axes of a plate are right-angled to each other. With this criterion, and the assumption that the thickness of the slab and its deflection are small, it is possible to apply a two-dimensional treatment. This produces a simplified expression because the elastic properties of third symmetry can be disregarded. Equation 12 and table 3.1 are modified for non-orthotropic plates and lead to a value for their natural frequency.

$$f = \frac{1}{2\pi}\sqrt{\frac{\dfrac{A^4 D}{a^4} + \dfrac{B^4 D}{b^4} + \dfrac{2CD'}{a^2 b^2}}{m}} \tag{12}$$

With

$$D' = \frac{E\,\nu\,h^3}{12\ (1-\nu^2)} + \frac{G\,h^2}{6} \tag{13}$$

boundary conditions	A	B	C	m	n
	4.730 4.730 β_2 β_2	4.730 ε_2 4.730 ε_2	151.3 $12.30\varepsilon_2(\varepsilon_2-2)$ $12.30\beta_2(\beta_2-2)$ $\beta_2\varepsilon_2(\beta_2-2)(\varepsilon_2-2)$	2 2 3,4,5,... 3,4,5,...	2 3,4,5,... 2 3,4,5,...
	4.730 β_2	ε_1 ε_1	$12.30\varepsilon_1(\varepsilon_1-1)$ $\beta_2\varepsilon_1(\beta_2-2)(\varepsilon_1-1)$	2,3,4,... 2,3,4,...	2 3,4,5,...
	4.730 β_2	ε_0 ε_0	$12.30\ \varepsilon_0^2$ $\beta_2\varepsilon_0^2(\beta_2-2)$	2 3,4,5,...	2,3,4,... 2,3,4,...
	β_1	ε_1	$\beta_1\varepsilon_1(\beta_1-1)(\varepsilon_1-1)$	2,3,4,...	2,3,4,...
	β_1	ε_0	$\beta_1\varepsilon_0^2(\beta_1-1)$	2,3,4,...	2,3,4,...
	β_0	ε_0	$\beta_0^2\ \varepsilon_0^2$	2,3,4,...	2,3,4,...

$$\beta_0 = (m-1)\pi \qquad \beta_1 = (m-0.75)\pi \qquad \beta_2 = (m-0.5)\pi$$
$$\varepsilon_0 = (n-1)\pi \qquad \varepsilon_1 = (n-0.75)\pi \qquad \varepsilon_2 = (n-0.5)\pi$$

Table 3.1 Frequency paramenter provided by Hearmon

3.2.6 Approximation Presented by Jänich

Another approximation that uses the Rayleigh method to solve the calculation of fundamental frequency is presented by Jänich, who modified the double integral of the potential as well as kinetic energy to gain a final equation for the fundamental frequency. This equation also considers additional loadings. Excepting static loads, this approximation makes it possible to include additional uniform loads (e.g. floor pavement or tiling), point loads or even imposed loads. These considerations are covered in an extra parameter determined by the kinetic energy and are given by:

$$N = N_0\left(1 + \frac{p}{g}\right) + \sum \frac{P}{g} w_P^2 \tag{14}$$

Where p is the additional load (coating + imposed load), g is the mass of a slab, P is the point load and w_p is the displacement below the point load P.

A second parameter was derived from the potential energy to get a uniform calculation. The K value consists of:

$$K = \frac{K_1}{a^4} + \frac{K_2}{a^2 b^2} + \frac{K_3}{b^4} \tag{15}$$

The necessary values for N_0, K_1, K_2 and K_3 can be taken from table 3.2, which shows an extract providing eight different examples of support condition and their corresponding parameters. The original table presented by Jänich includes 18 different set-ups of boundary condition with combinations of free, clamped and simply supported edges.

With both parameters, N and K, and the equation presented below, the first natural frequency may be estimated.

$$f_0 = \frac{\pi}{2}\sqrt{\frac{D\, g\, K}{h\, \gamma\, N}} \tag{16}$$

Another advantage of this method is its capability of estimating the frequencies of two-span slabs. Provided that both jointed edges of a slab have equal support conditions, equation 16 can be modified to estimate the fundamental frequency with:

$$f_0 = \frac{\pi}{2}\sqrt{\frac{D\,g\,(\sum a_i^2 K_i)}{h\,\gamma\,(\sum a_i^2 N_i)}} \tag{17}$$

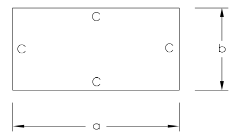
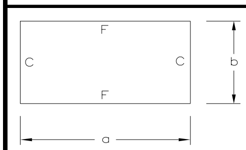
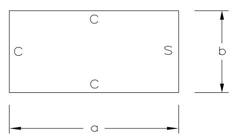
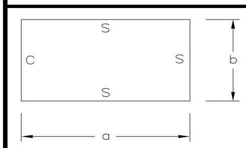
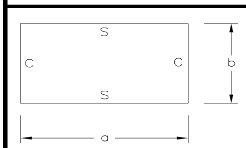
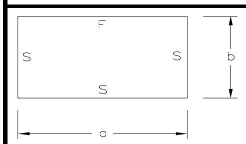
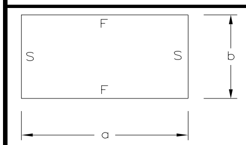
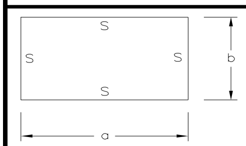
boundary condition	K_1	K_2	K_3	N_0
	12.00	8.00	12.00	2.25
	8.00	0.00	0.00	1.50
	3.84	5.00	8.00	1.50
	1.28	1.25	0.50	0.50
	4.00	2.00	0.75	0.75
	0.1667	0.0760	0	0.1667
	0.50	0	0	0.50
	0.25	0.50	0.25	0.25

Table 3.2 *K* and *N* parameters

3.2.7 Estimation for Pin Supported Plates

Reed Jr. (1965) published a NASA report in which he presented and compared two different approaches to calculating the natural frequency of rectangular plates supported by isolated pins in each corner. The two approximate methods were then developed using the Ritz method and a series solution to the differential equation of motion. The general solution for this type of support can be gained by treating a plate as those with supports along their entire perimeter. Afterwards superpositions of the initial solutions must be done to satisfy the specific conditions of the isolated pin supports.

Comparing both methods revealed a higher accuracy for the series solution but also more difficulties in computing this approach. However, because of the need for simplified hand estimation and the already existing frequency parameter, this disadvantage can be disregarded here.

With both methods the natural frequency can be obtained by:

$$f = \frac{\lambda \cdot \pi}{2a^2} \left[\sqrt{\frac{\gamma \cdot h}{D \cdot g}} \right]^{-1} \tag{18}$$

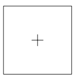
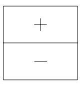
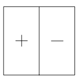
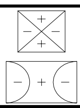
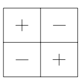
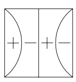
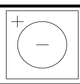
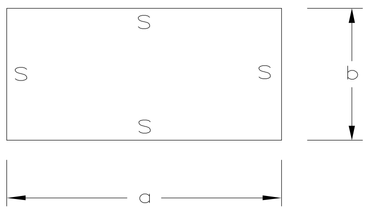
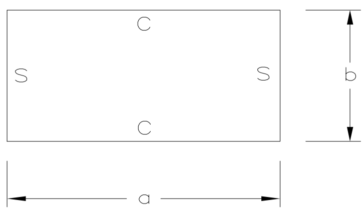
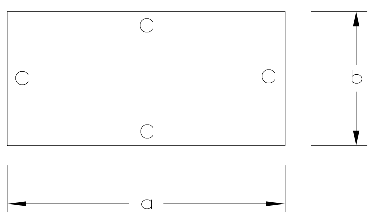
Mode		α	frequency paramenter λ	
			Ritz solution	Series solution
1		1.0 1.5 2.0 2.5	0.756 0.933 0.958 0.961	0.721 0.904 0.941 0.951
2		1.0 1.5 2.0	1.702 2.308 2.941	1.598 2.181 2.786
3		1.0 1.5 2.0	1.702 2.811 3.52	1.598 2.616 3.326
4		1.0 1.5 2.0	1.986 3.53 5.69	1.986 3.414 5.27
5		1.0 1.5 2.0	4.2 5.67 6.8	3.895 5.34 6.46
6		1.0 1.5 2.0	5.23 5.85 7.4	5.1 5.85 7.22
7		1.0 1.5	4.89 7.64	4.5 7.1

Table 3.3
Frequency parameters for pin supports

3.2.8 Compilation of Formulas by Bachmann

Walter Ammann and Hugo Bachmann, one of the most well-known authors in the field of floor vibration issues, provided in their book a set of tables and charts for estimating the natural frequencies of several structural systems. These were informed by the partial differential equation of motion for free vibration. All plates considered in these tables are assumed to be two-way spanning and have boundary conditions of simply supported, clamped or free edges. One extract of these tables is given in table 3.4. It includes simply supported, clamped and a mix of both conditions and allows the calculation of the first two natural frequencies of these samples.

$$f_i = \frac{\varphi_i}{a^2}\sqrt{\frac{\mathrm{E}\cdot \mathrm{h}^3}{12\,(1-\nu^2)\,m}} \tag{19}$$

support conditions	
	$\varphi_{1,1} = 1.57\ (1+\alpha^2)$ $\varphi_{1,1} = 6.28\ (1+0.25\alpha^2)$ $\varphi_{1,1} = 1.57\ (1+4\alpha^2)$
	$\varphi_{1,1} = 1.57\sqrt{1+2.5\alpha^2+5.14\alpha^4}$ $\varphi_{2,1} = 6.28\sqrt{1+0.625\alpha^2+0.321\alpha^4}$ $\varphi_{1,2} = 1.57\sqrt{1+9.32\alpha^2+39.06\alpha^4}$
	$\varphi_{1,1} = 1.57\sqrt{5.14+3.13\alpha^2+5.14\alpha^4}$ $\varphi_{2,1} = 9.82\sqrt{1+0.298\alpha^2+0.132\alpha^4}$ $\varphi_{1,2} = 1.57\sqrt{5.14+11.65\alpha^2+39.06\alpha^4}$

$$\varphi_1 = \varphi_{1,1}\ ;\ \varphi_2 = \min\begin{Bmatrix}\varphi_{1,2}\\ \varphi_{2,1}\end{Bmatrix}$$

Table 3.4 Frequency paramenter provided by Bachmann

For the case of continuous plates a chart is presented in figure 3.1, which helps to estimate the first three natural frequencies of a two-span slab. However, because this chart was developed for continuous beams, it is only applicable to one-spanning slabs. One indication of this limitation is the simple assumption of parameters in equation 20 without reference to plate characteristics such as the plate rigidity or the width of the slab.

$$f_n = \frac{\lambda_n}{2\pi}\left[\sqrt{\frac{m}{EI}}\right]^{-1} \tag{20}$$

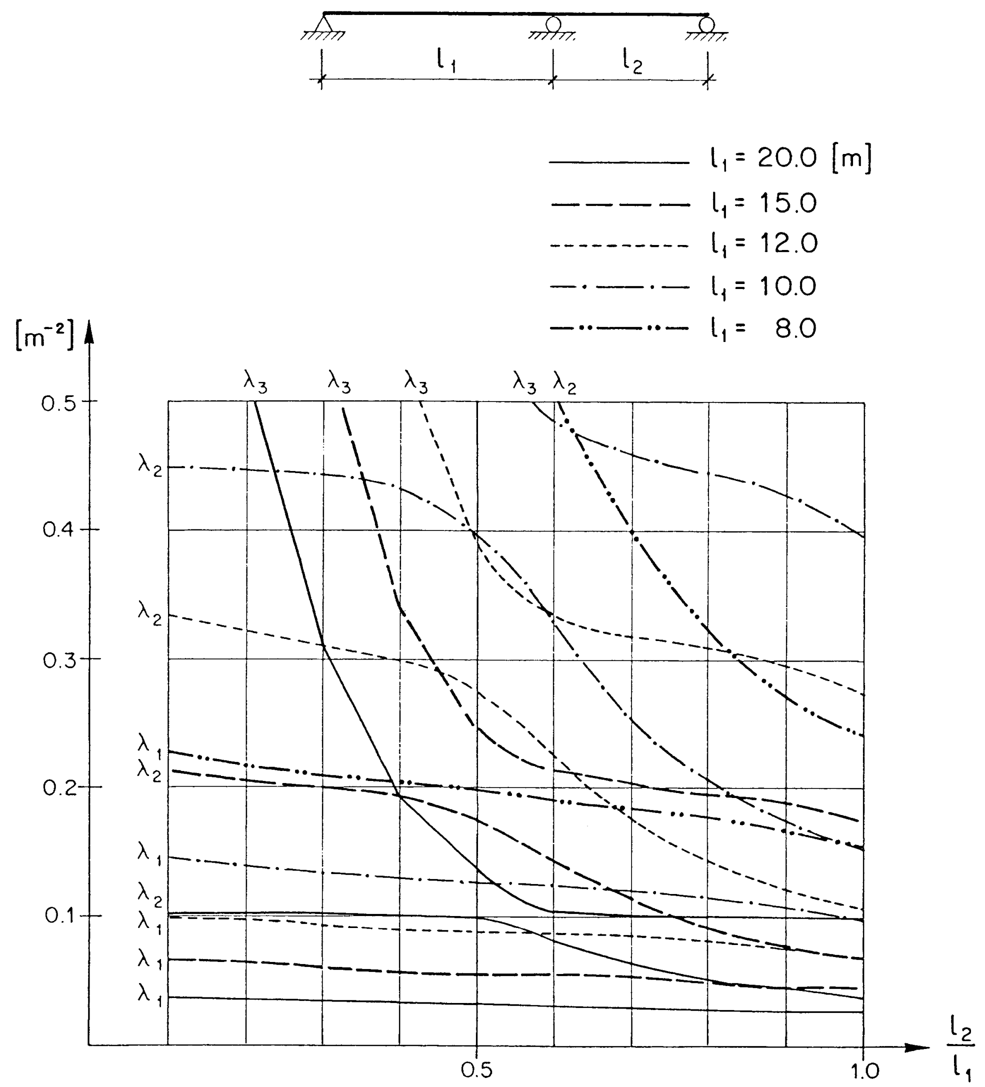


Figure 3.1 Frequency parameter for continuous slabs

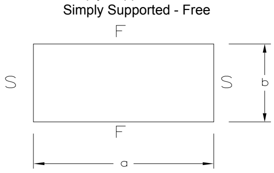
3.2.9 Compilation of Formulas by Blevins

Robert D. Blevins (1979) published a huge number of different sets for simplified hand calculation in *Formulas for Natural Frequency and Mode Shape*. The book is intended as a reference for engineers and provides tables for many different structures and shapes. It also considers different boundary conditions and diverse mode shapes. The tables are compiled from a variety of sources. In the case of rectangular plates (Page 252-278) the layout is similar to that previously considered with the addition of pin supported slabs.

The natural frequency can be estimated by:

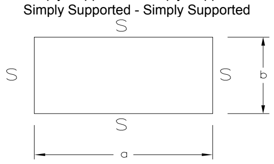
$$f_i = \frac{\lambda_i^2}{2\pi \cdot a^2} \left[\frac{E \cdot h^3}{12 \cdot m \cdot (1 - \nu^2)} \right]^{1/2} \tag{21}$$

Simply Supported - Free - Simply Supported - Free



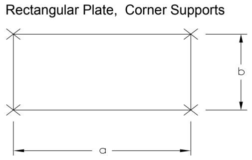
$^a/_b$	frequency parameter λ_i^2					
	1. Mode	2. Mode	3. Mode	4. Mode	5. Mode	6. Mode
0.4	9.760	11.040	15.060	21.710	31.180	39.240
2/3	9.698	12.980	22.950	39.110	40.360	42.690
1	9.631	16.140	36.730	38.950	46.740	70.740
1.5	9.558	21.620	38.720	54.840	65.790	87.630
2.5	9.484	33.620	38.360	75.200	86.970	130.400

Simply Supported - Simply Supported - Simply Supported - Simply Supported



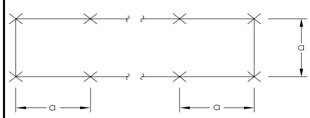
$^a/_b$	frequency parameter λ_i^2					
	1. Mode	2. Mode	3. Mode	4. Mode	5. Mode	6. Mode
0.4	11.45	16.19	24.08	35.14	41.06	45.80
2/3	14.26	27.42	43.86	49.35	57.02	78.96
1	19.74	49.35	49.35	78.96	98.70	98.70
1.5	32.08	61.69	98.70	111.00	128.30	177.70
2.5	71.56	101.16	150.50	219.60	256.60	286.20

Rectangular Plate, Corner Supports



$^a/_b$	λ_1^2	λ_2^2
1	7.12	15.80
1.5	8.92	21.50
2	9.29	27.50
2.5	9.39	35.50

N-Bay Plate



Number of Bays	λ_1^2	λ_2^2	λ_3^2
1	7.18	16.30	16.30
2	16.27	16.76	33.28
3	24.41	25.41	28.39
4	33.02	33.41	37.20
5	41.41	41.86	45.43

1 bay = square plate

Table 3.5 Frequency paramenter provided by Blenvis

3.3 Analysis of Results

The fundamental frequencies of 12 different Cobiax flat slabs were estimated by simplified hand calculation. Afterwards each solution was compared with its corresponding value calculated with finite element software. A summary including all methods and all calculated examples is contained in Appendix A, which also provides the ratio of frequency derived from simplified calculations to an exact finite element solution for each example.

The evaluation of accuracy is informed by figure 3.2 where frequencies estimated with simplified hand calculations are plotted on the x-axis and the solution from FEM on the y-axis. Each marking represents one calculated value with its particular method. The dashed line stands for the FEM values and symbolises the ideal position for the calculated values. The closer a point is located to the 'FEM line' (vertical or horizontal distance), the higher its accuracy compared to the finite element solution.

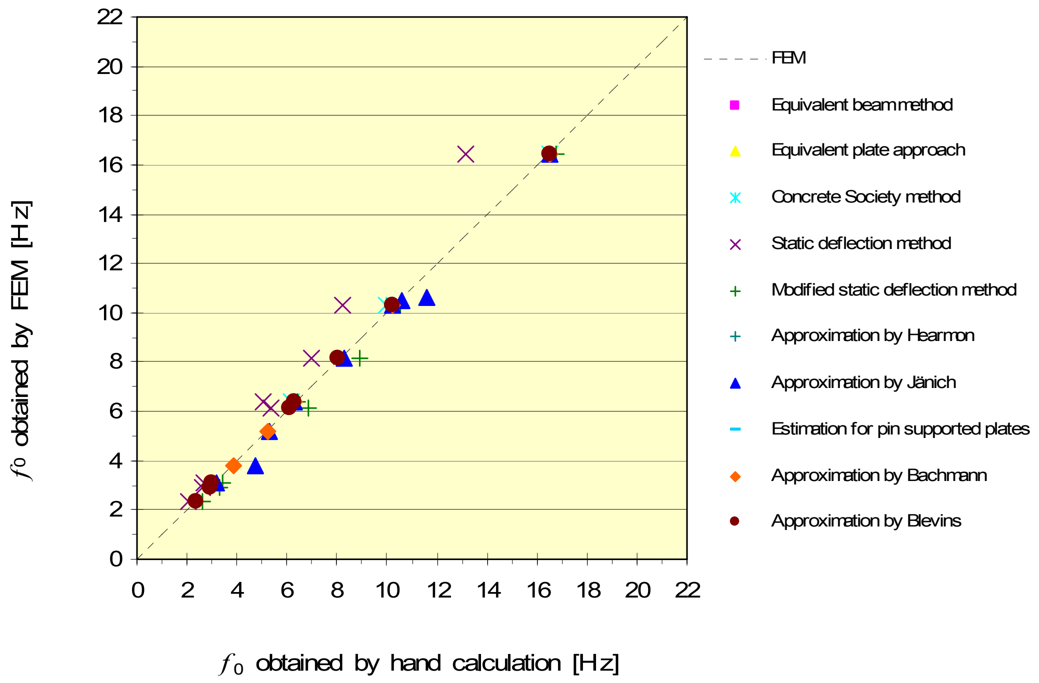


Figure 3.2 Comparison of hand-calculated and computed frequencies

Overall, there is a reasonably good correlation between the hand-calculated approximations and the computed values. Nevertheless it is noticeable that some values have higher inaccuracies. The static deflection method (magenta cross) is the most inaccurate estimation method, underestimating all its frequencies. In contrast, the modified static deflection method (green plus sign) tends to overestimate its solutions. This becomes clearer if the ratio of approximation to computed value is plotted for each example, as in figure 3.3, making it possible to see their higher variation in comparison with all other methods. The fact that both methods consist of the same parameters and only differ in a modification factor explains their similar pattern.

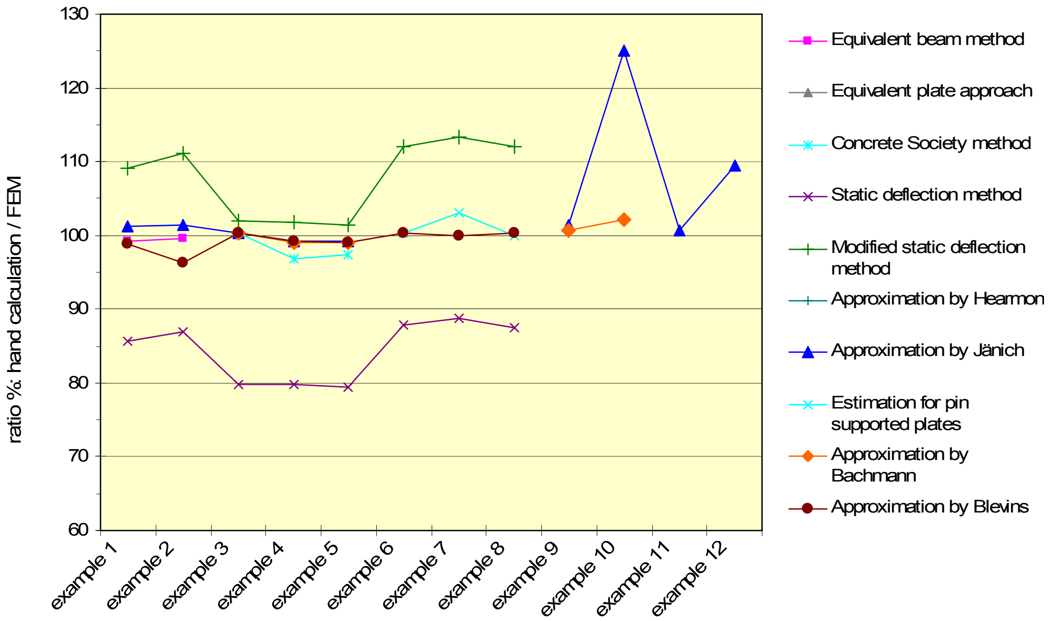


Figure 3.3 Individual accuracy of approximations

This graph also clarifies the higher differences of Jänisch's method (blue triangle) for example 10 (25%) and example 12 (9.4%). As these examples involve continuous slabs with two varying span lengths, it might be supposed that this approximation is more qualified for continuous slabs with equal span length. This assumption is confirmed by examples 9 and 11 calculated using Jänisch's method, which fulfil this condition and have variances of only 1.5% and 0.6%.

3.4 Conclusion

Ten different methods for estimating the fundamental frequency of concrete floors have been presented. The general aim has been to provide an overview of their accuracy and therefore their serviceability for an initial estimation. Their integrity was checked by comparing the solutions of concrete examples with accurate values calculated with finite element software. Although some variations occurred among each method, all methods provided good predictions and suffice for an initial assessment. It should always be considered that these methods are only approximations and are conducive to rather than conclusive in evaluating slabs' natural frequency.

The average ratio of hand calculated values to computed values is plotted for each respective estimation method in figure 3.4, which indicates the common high accuracy of all methods while also confirming the relative inaccuracy of the static deflection and modified static deflection methods.

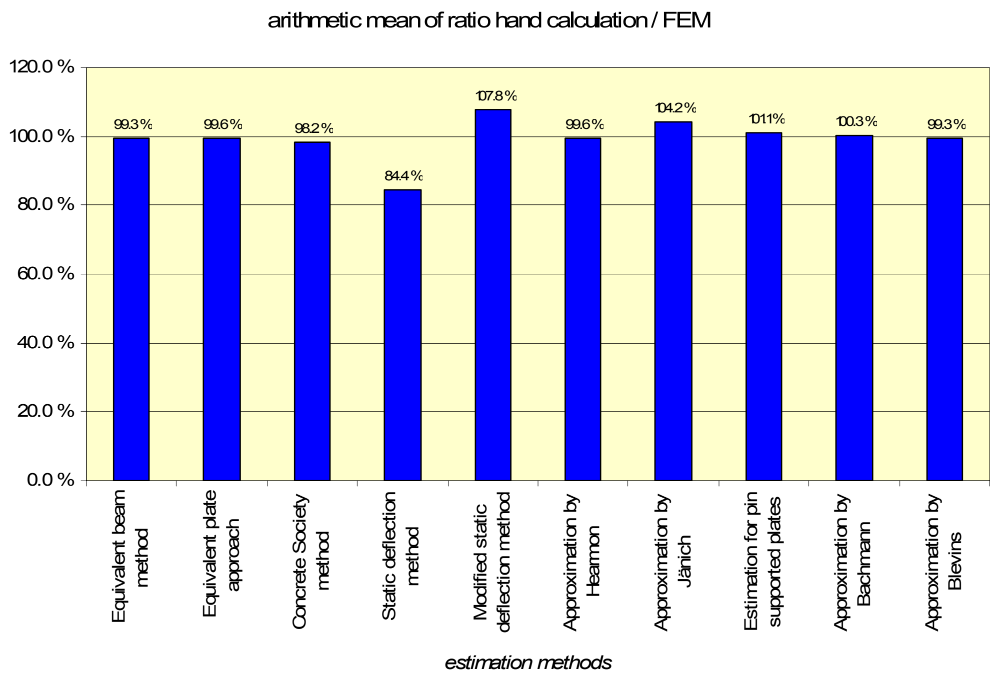


Figure 3.4 Average accuracy of approximations

This inaccuracy could occur as a result of these methods' general application. In contrast to all other tested approximations specified for one fixed set-up, both the deflection methods may be used for all kinds of boundary condition. An even more important factor for their less accuracy is their derivation, as shown in 3.2.4. Because their origin is derived form the equation of a one-degree-of-freedom system considering just one mass, it is an extremely approximation. However, even with variations of 15.6% and 7.8%, the estimated values still can be used for a rough prediction of vibration performance.

The use of adapted equations for specific boundary conditions provides very close values compared to a finite element solution. Methods considering different kinds of slab parameters in the present investigation yielded an accuracy of 4.2% (overestimation) and 1.8% (underestimation), results more than good enough for an approximate assessment of the fundamental frequency of slabs.

4. Numerical Analysis

The specific qualities of a Cobiax flat slab as compared to a traditional solid slab include a decreased stiffness and mass. As these parameters are two main factors influencing natural frequency, this change has an impact on vibration performance. However, because of the contrary effect of the decreased values it is as yet undetermined how the final results are affected. An investigation will be performed to clarify this lack of knowledge.

A series of detailed investigations will be carried out to evaluate the specific behaviour of Cobiax slabs in relation to natural frequency. Calculations with finite element software will be undertaken considering a whole range of common situations in the designs of floors. Parameters such as geometry, boundary conditions and loadings will change for each example. This variety guarantees the feasibility of an overall and universally valid assessment. To evaluate the Cobiax flat slab system, every example is also carried out for conventional solid slabs which allows for a comparison of the slab types and assesses the quality of Cobiax slabs in these conditions.

4.1 Software

Because this research was undertaken in collaboration with Cobiax Technologies GmbH, Germany, special software was provided for the investigation, namely the finite element software Tornow-Software, established in Germany since 1983. The software is subdivided into several packages for individual scopes. For this investigation the “FEM-Tripla” package, developed for the design of floor systems, was applied. A big advantage here was its additional ‘Cobiax module’ (Figure 4.1), specially generated for the design of Cobiax flat slabs. After the input of all necessary data, including thickness of the slab and ball diameter, the corresponding decreasing of mass and stiffness are taken into consideration.

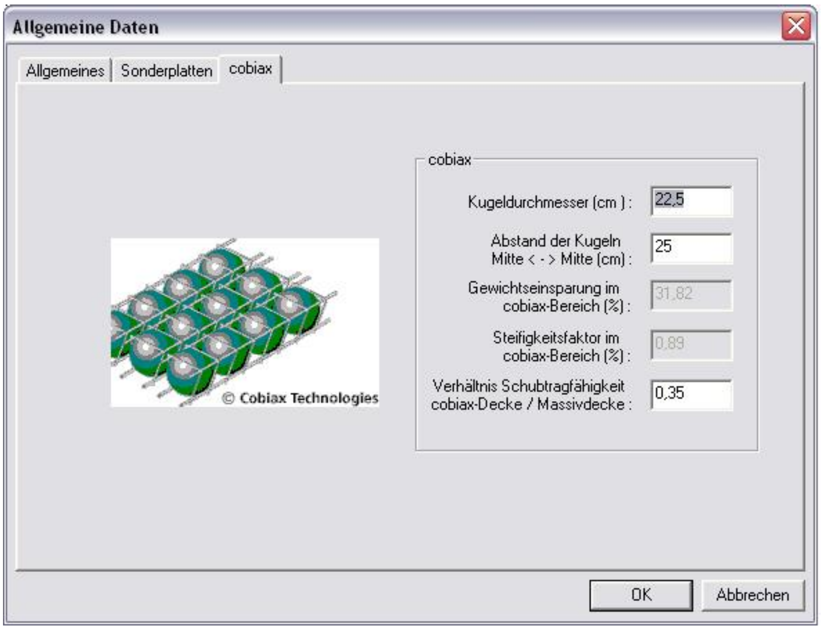


Figure 4.1 Cobiax module

The package has different set-ups for investigating natural frequency and is capable of calculating up to 10 natural frequencies using different approaches. The following investigation considers the first three frequencies with the main focus on fundamental frequency. Analysis of the natural frequencies is calculated using the Lanczos algorithm, an iterative algorithm which employs the Lanczos recursion. This is a process of defining or expressing a function or the solution to a problem in terms of itself, by producing a recursive function. However, it is also a very powerful solver and well known as an efficient method of finding eigenvalues and eigenvectors of matrices. The Lanczos procedure is generally used for large sparse matrices. Cullum and Willoughby (1985) summarised the basic steps in any Lanczos procedure as shown below.

1. Transform a given 'symmetric' matrix A into a family of 'symmetric' tridiagonal matrices of varying sizes
2. Compute eigenvalues and eigenvectors of certain members of this family
3. Take some or all of these eigenvalues as approximations to eigenvalues of matrix A and map the corresponding eigenvectors of the tridiagonal matrix into Ritz vectors for matrix A
4. Use these Ritz vectors as approximations to the eigenvectors of A

The accuracy of the calculated frequencies is set to 10^{-5} and is also improved by a small and detailed mesh.

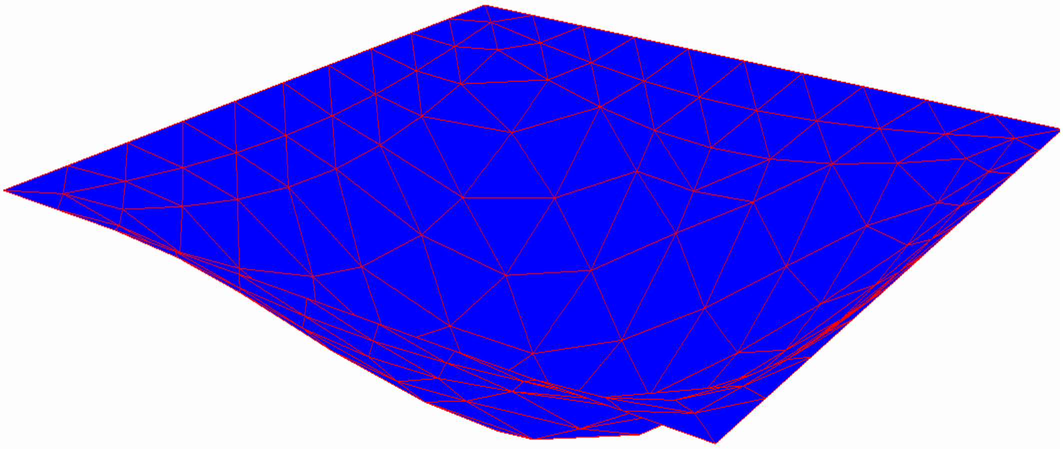
4.2 Verification of Software Accuracy

In terms of the obtained values' accuracy, an initial test is carried out. A proof in form of a comparison between Tornow-Software and a second finite element software will clarify that Tornow-Software provides exact and accurate values. For this purpose a slab was modelled with both types of software and its fundamental frequency as well as its mode shape was calculated.
The used software is called RFEM 2.01 established by Dlubal Software. It is a finite element software applicable for a wide range of tasks in structural engineering.

The comparison consists of an example considering following parameters:

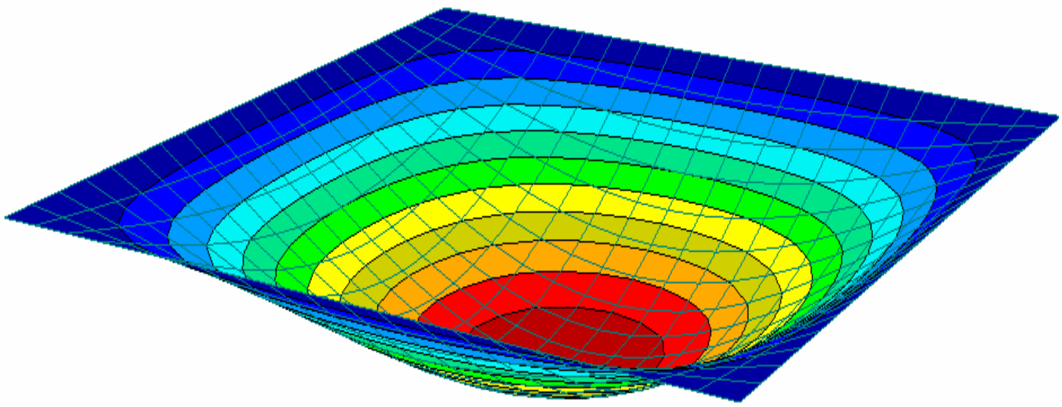
- One span solid slab
- All edges are simply supported
- 10m x 10m x 0.3m
- E = 28,300 N/mm² ; ν = 0.2 (C30/37)
- Imposed load q = 5.0 kN/m²

The results gained of both calculations are presented in figure 4.2 and figure 4.3 below.



Fundamental frequency f_0 : 7.135 Hz

Figure 4.2 1.Mode shape obtained by Tornow-Software



Fundamental frequency f_0 : 7.148 Hz

Figure 4.3 1.Mode shape obtained by RFEM

The conclusion of this comparison is unambiguous. Both estimated fundamental frequency are nearly the same value. Only a very little difference of less than 0.2% is realised which could be influenced by differences in meshing the slab or different calculation approaches.
Nevertheless, this initial comparison leaves no doubt in the exactness of values described in this chapter. Furthermore the good correlation between computed vales used Tornow-Software and hand-calculated vales in chapter 3 is an additional indicator of the well performed accuracy of Tornow-Software.

4.3 General Settings

The most important factors in the provision of a correct evaluation are realistic and comparable values. Some initial settings were carried out with the intention of ensuring these necessary conditions. By applying common material properties, geometries and construction forms which are used in practice the need to be realistic is satisfied. As far as the comparability requirement is concerned, constant estimations throughout the entire investigation will provide a good solution.
The investigation includes seven common types of slabs in construction:

Example 1: Simply supported slab, one-way spanning
Example 2: Simply supported slab, two-way spanning
Example 3: 2 span slab, two-way spanning
Example 4: 3 span slab, two-way spanning
Example 5: 1x1 bay slab, supported by columns
Example 6: 2x1 bay slab, supported by columns
Example 7: 3x3 bay slab, supported by columns

All line supports used in this investigation are considered to be simply supported without any restraints. Columns modelled in examples 5 – 7 are assumed to be pinned, also without any restraints.

For each of these seven systems a range of different geometries are regarded. These include changes in length of spans (6m-17m) and different width of spans (4m-17m). The proper thickness was obtained with information offered on Cobiax's website (2006) and is provided in appendix C. One good resource here was a diagram presenting amongst other things the interrelationship between slab length and necessary thickness as well as ball diameter. Furthermore, a list of already existing projects including floor geometries and ball diameters was used to estimate an appropriate deck thickness. Depending on this thickness, which ranges from between 30cm and 60cm, a proper Cobiax hollow sphere (Ø22.5cm-Ø45cm) was used. A detailed description including all parameters is given in the corresponding tables in appendix B.

As regards material properties, realistic terms are assumed by using a concrete providing a quality of C30/37. As this is an averaged concrete quality, common in design, it will deliver useful values. Referring to DIN1045-1, 9.1.7, its mean Young's modulus is 28,300 N/mm². The second material quality, the Poisson ratio, is supposed to be 0.2 for the concrete used in both types of slabs. According to DIN 1055-1, 5.1, the unit weight for reinforced concrete is 25.0 kN/m³, the value used to calculate the dead load. In addition, each slab is also loaded with 1.5 kN/m² to represent loading due to possible use of coating.

For further loadings it is important to consider the wide range of structures Cobiax flat slabs are suitable for, including residences, offices and car parks. Thus, rather than limiting this investigation to one design situation, as wide an application field as possible is regarded. The imposed loads are staged according to this purpose. A loading of 5.0 kN/m² is stepwise decreased by degrees of 25%, leading to further loads of 3.75 kN/m², 2.50 kN/m², 1.25 kN/m², and finally 0 kN/m². In addition to the dead load these five different loadings are used for each slab, providing a variety of possible set-ups.

4.4 Analysis of Results

The overall result of this investigation is that for all types of slabs, including their entire range of varying dimensions, Cobiax flat slabs reached higher and therefore better natural frequencies than traditional solid slabs with the same geometries. The absolute values range from 0.022 Hz (2x1 bay slab, 30cm, 5.0 kN/m²) to 3.813 Hz (2 span slab, 30cm, 0 kN/m²).

The trend of fundamental frequencies for each example of a single span, two-way spanning slab is plotted in figure 4.4. To present a better view, only two load situations (0 kN/m² and 5.0 kN/m²) are displayed. It is noticeable that by increasing the applied load, the difference of the absolute values for natural frequency between Cobiax slabs and solid slabs decreases but, as will be shown shortly, their ratios stay constant. Furthermore a decreasing and simultaneously a narrowing of frequencies is observable if the slab systems become larger.

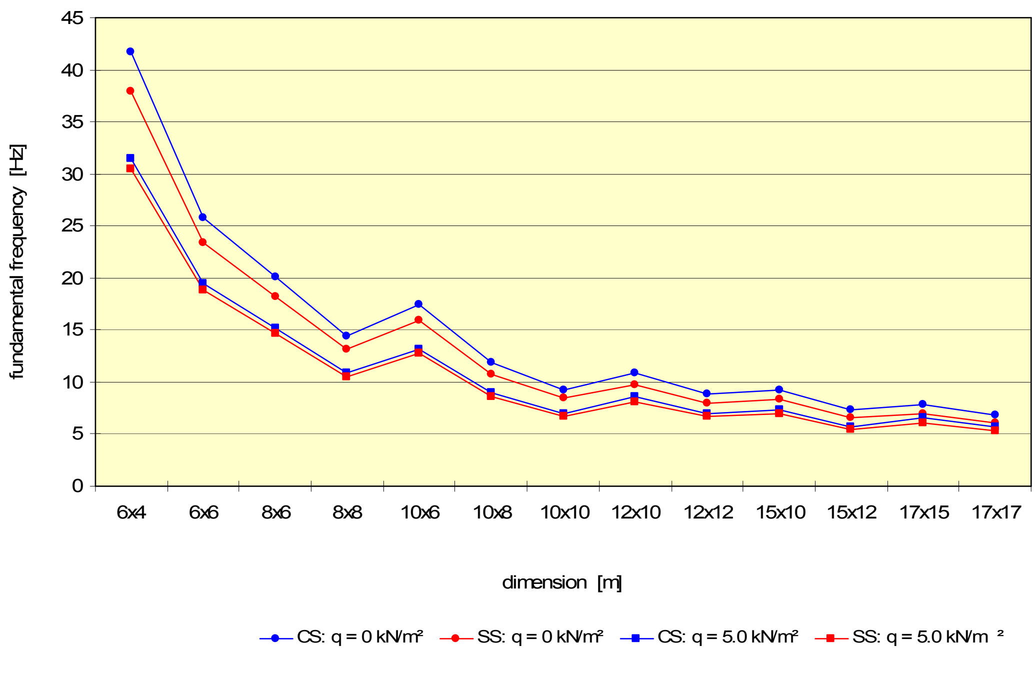


Figure 4.4 Fundamental frequencies for a single span slab

With larger spans, the natural frequency becomes reduced for both types of slabs, resulting in smaller differences between the values. This is explained by the constant ratio of frequency of Cobiax slabs to solid slabs. Provided that the slab thickness and ball diameter of a Cobiax slab stay constant, the reduction of stiffness and mass will not change either. This leads to a specific and constant value for each slab in which both types differ regardless of their geometry. This means that the ratio f_{cs}/f_{ss} of a 6m x 4m slab is equal to any other slab dimension, as long as its deck is 30cm thick and a sphere with a diameter of 22.5cm is included.

If the relative values are regarded, it is not necessary to consider all examples of a slab but only its different thicknesses. This leads to an improved view of the results as shown in figure 4.5. The relationship between deck thickness, applied loads and the resulting difference in fundamental frequency is plotted. This consideration has the advantage of allowing the possibility of evaluating vibration performance comparatively.

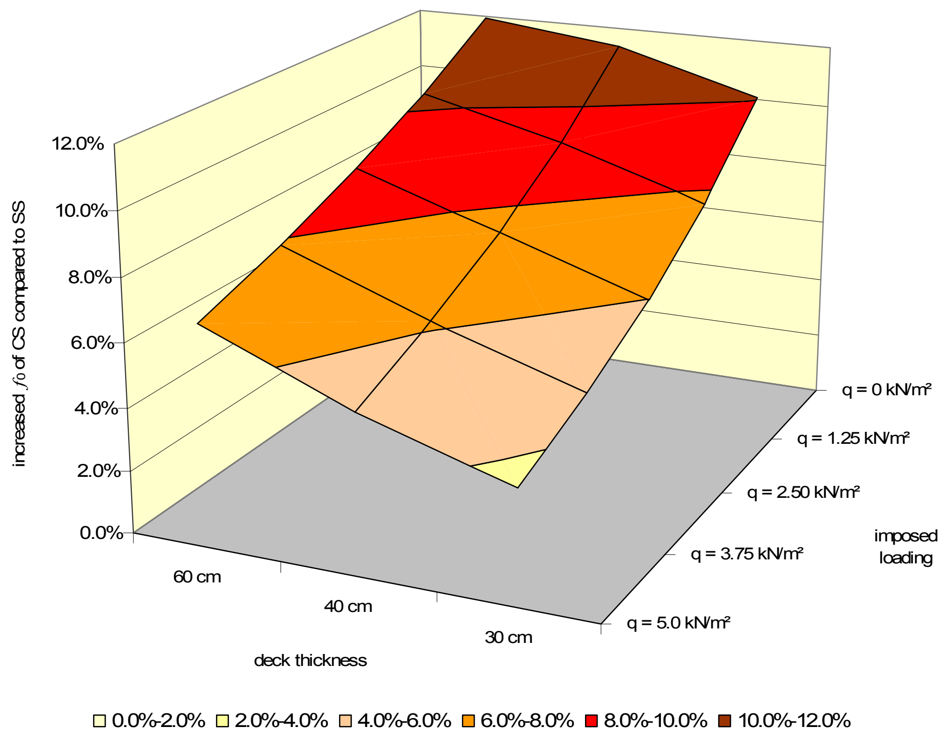


Figure 4.5 3-D view of frequency dependency

The three-dimensional shape provides information on how the vibration behaviour of Cobiax flat slabs as compared to conventional slabs improves by increasing load and thickness. The sloped surface with increasing gradients towards higher thicknesses as well as lesser loadings indicates that Cobiax slabs increase their advantage of higher frequencies for these two changes. In the best case, when a 60cm thick floor is loaded according only to its own self-weight, the fundamental frequency of a Cobiax slab is 11.9% higher than its solid equivalent. Its minimum advantage of 3.6% is obtained from the Cobiax systems for a 30cm deck with a 5.0 kN/m² applied load. These two values define the range along which all others comparisons are located. An extensive summary of all differences is shown in table 4.1.

	30cm (Ø22.5cm)	40cm (Ø31.5cm)	60cm (Ø45cm)
q = 0 kN/m²	10.1%	11.4%	11.9%
q = 1.25 kN/m²	7.7%	9.2%	10.3%
q = 2.50 kN/m²	6.0%	7.5%	9.0%
q = 3.75 kN/m²	4.6%	6.2%	7.9%
q = 5.0 kN/m²	3.6%	5.0%	7.0%

Table 4.1 Cobiax' advantage related to loads and thickness

The values of this table are also plotted in the shape of smoothed curves in figure 4.6. In addition to the latter figure and table these three curves and therefore the predominance of Cobiax slabs also clarify their decrease with incremental loading. The point of intersection of the curves and the x-axis is worthy of consideration. This point would provide the amount of loading at which Cobiax slabs will achieve the same natural frequencies as solid slabs. For any point below the ratio would change and the solid slabs would gain higher natural frequencies.

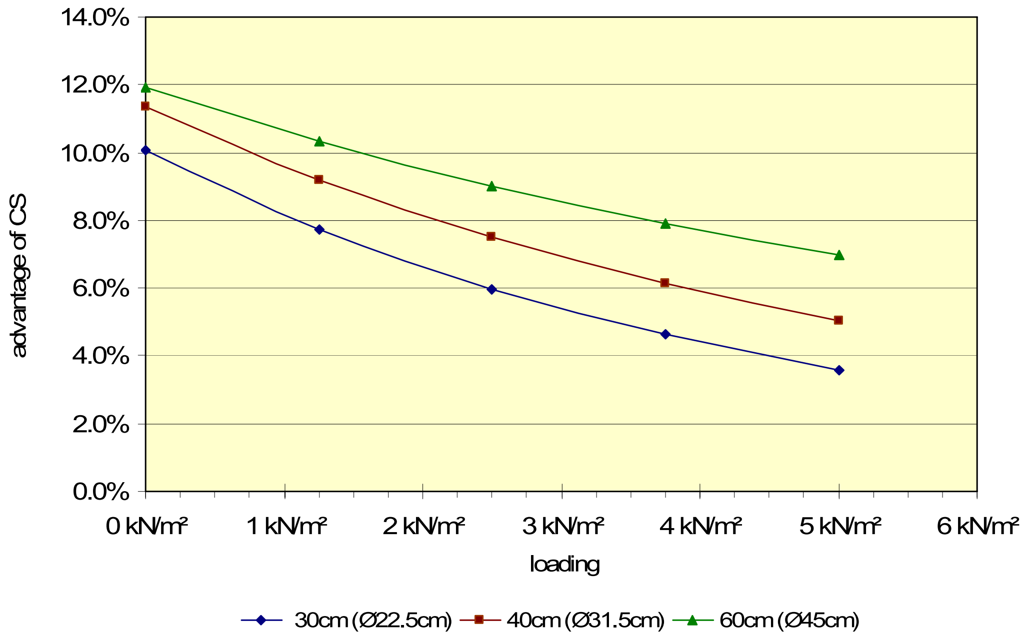


Figure 4.6: Cobiax advantages against loading

If an approximation is applied, it is possible to express the falling trend with the help of a cubic equation able to deliver knowledge concerning the further run. Afterwards these equations can be used to calculate the approximate intersection of curve and x-axis. This will be the boundary for which Cobiax flat slabs have an improved vibration behaviour compared to solid floors.

h = 30cm : $y = -0.0001x^3 + 0.0024x^2 - 0.0215x + 0.1005$

h = 40cm : $y = -0.0001x^3 + 0.0019x^2 - 0.0195x + 0.1137$

h = 60cm : $y = -4*10^{-05}x^3 + 0.001x^2 - 0.0137x + 0.119$

These equations yield maximum values for uniform loadings of (including
1.5 kN/m² due to coating):

h = 30cm : max. q = 15.1 kN/m²

h = 40cm : max. q = 12.2 kN/m²

h = 60cm : max. q = 16.8 kN/m²

Again, this can be expressed by ratios of applied load to self-weight:

h = 30cm : q/g = 2.95

h = 40cm : q/g = 1.83

h = 60cm : q/g = 1.64

These values are valid for all Cobiax slabs with thicknesses of 30cm, 40cm or 60cm including a sphere diameter of 22.5cm, 31.5cm or 45cm, regardless of their boundary condition.

Because these values are gained by extrapolated calculations, a confirmation is necessary. For this purpose additional finite element calculations were carried out, including the 'critical' loadings. All examples showed very good high similarity between both fundamental frequencies.
Exemplary table 4.2 present results for a one span slab simply supported on all four edges. With the utmost probability, the slightly variance of these numbers occur due to the approximation during the extrapolation procedure.

Dimension	CS f_0 [Hz]	SS f_0 [Hz]	ratio f_{CS}/f_{SS}
8m x 8m x 0.3m	8.279	8.299	0.998
12m x 12m x 0.4m	5.843	5.740	1.018
17m x 17m x 0.6m	4.491	4.393	1.022

Table 4.2 Accuracy of 'critical' load for one span slab

Another check of the accuracy is given in figure 4.7. This chart has plotted the direct relationship of applied load and resulting fundamental frequency for both types of slab. In this case, the values relate to a continuous slab with two equal spans and the same length-width ratio. Like before, it is noticeable that the obtained 'critical' value for 30cm slabs provides the highest accuracy which is indicated by the very close intersection of both upper curves relating to the predicted value (dashed line). But even if the approximation according to 40cm and 60cm slabs are less accurate, the variance of 2% is still accurate enough to evaluate the relationship between both types of slab.

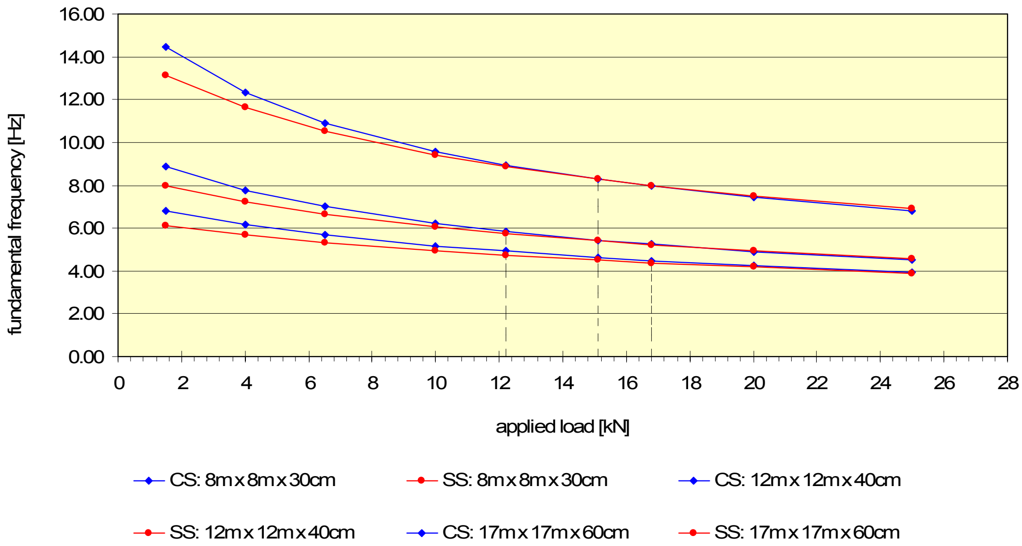


Figure 4.7 Accuracy of 'critical' load for continuous slab

The change in ratio between the different types of slab is caused by the decreasing relevance the Cobiax slabs' reduced self-weight. If the applied loadings are similar to the self-weight, the reduction of approximately 30% mass is an advantage for Cobiax slabs. However, once the applied loads increase, the self-weight becomes just a small amount of the overall load and is therefore negligible. If this occurs, the only difference on the part of Cobiax slabs is the reduced stiffness which has a negative impact on the vibration behaviour and therefore leads to lower natural frequencies.

4.5 Conclusion

An evaluation of Cobiax flat slabs (CS) compared to traditional solid slabs (SS) was required. For this purpose an investigation of 940 different slabs including changes in type (CS/SS), dimensions and boundary condition was carried out.

The overall conclusion is that Cobiax flat slabs possess higher fundamental frequencies for all investigated combinations. However, it was also presented that this only occurs if a specific load to self-weight ratio exists. Indeed, Cobiax slabs lose their advantage of mass reduction after a certain point; but due to the amount of loadings required to realise this, its general performance is not affected. More precisely, the usual fields of application for Cobiax slabs are offices, public buildings and car parks which could all be expected to bear a typically lesser imposed load than the 'critical' values. Table 4.3 shows some ranges of loadings which should be considered in these areas according to the British as well as German Standard.

Type of structure	Uniformly distributed load [kN/m²]	
	BS 6399-1 (1996)	DIN 1055-3 (2002)
Residence	1.5 - 4.0	1.5 - 2.0
Office and similar use	2.0 - 5.0	2.0 - 5.0
Public areas	2.0 - 7.5	3.0 - 5.0
Car parks (vehicles ≤ 25 kN)	2.5	2.5 - 5.0

Table 4.3 Extract of imposed loads in BS 6399-1 and DIN 1055-3

The provided values in both national Standards are all less than the minimum 'critical' value of $q = 12.2$ kN/m² for a 40cm thick Cobiax slab. For this reason the Cobiax system is supposed to gain higher natural frequencies than conventional solid slabs in all its projects.

5. Conclusions and Recommendations

5.1 General Conclusions

This research presents an overall view of natural frequency of concrete slabs in structural engineering. Although it is a very large field of consideration, already existing literature and research done in past provide good knowledge.

In case of human acceptance due to floor vibration a variety of recommendation is available. By people like Reiher and Meister (1931), Wiss and Parmelee (1974) or Brownjohn (2001) many investigations were carried out to improve the knowledge of this topic. Nowadays, with help of the wide range of recommendation, no significant complaints should occur. This conclusion is supported by different case studies including annoying floor vibration, as shown by Bachmann (1992) and Hanagan (2005). All cases describing vibration complaints confirm very low natural frequencies of their floors and therefore a high risk of perceptible motions.

Approximate hand-calculations presented in chapter three are a very well-know area. Due to the fact that computer supported calculations were not available, natural frequencies had to be estimated by hand calculations in the past. The high amount of existing literature gives an easy access and considers common slab types. In terms of accuracy of these simplifications performed comparisons of hand-calculated values and solutions obtained by finite element software confirmed their high quality. Even if some methods include small inaccuracy, it is still possible to use them as an initial estimation. With all existing computer software nowadays these methods are used for rough estimations anyway.

Chapter four containing the main issue of this research shows the improved vibration performance of Cobiax flat slabs compared to conventional solid slabs. A detailed investigation clarified the different effects of the reduced weight and stiffness of Cobiax slabs. It is shown that due to their lower dead load Cobiax flat slabs achieve higher natural frequencies for common practical use. However, with an increasing of imposed load this advantage decreases and after a certain point

Cobiax slabs show lower and therefore worse natural frequencies. This change happens because the 'negative reduction' of stiffness remains constant while after increasing the ratio of applied load to self weight the 'positive reduction' of mass decreases constantly until it becomes negligible.
An estimation of these 'critical' values indicates that because of the high amount of imposed loadings necessary to achieve this change, the vibration behaviour is still passable. All application areas Cobiax flat slabs used to focus show less imposed load than the 'critical' values obtained.

5.2 Areas of Future Research

Because the fact that vibration performance of structures covers very large field of consideration, further investigation are possible.

According to Lenzen (1966) one further important factor is the damping. A high damping value is necessary to reduced floor vibration during its first cycles to avoid human perception. Due to its reduced weight, the damping ratio is changed and leads to other differences between Cobiax slabs and traditional solid slabs.

Another point which should be regarded is the thickness variety of Cobiax slabs. Because each sphere diameter may be used for different thicknesses, the load reduction changes as well. Again, this leads to individual 'critical' values for each slab-thickness ratio. If, for example, a 22.5cm diameter ball is located inside a 40cm slab instead of a 30cm, the load reduction decreases from approximate 32% to 24%. According to chapter four this would decrease the 'critical' value at which Cobiax slabs lose their advantage compared to solid slabs.

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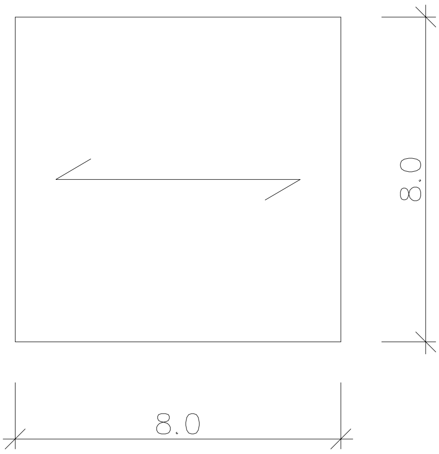
APPENDIX A

General Assumption

The accuracy of the simplified hand calculation method proposed in chapter 3.2 was checked by the following example. To obtain comparable values, all methods within each example dealt with the same set-up. It was assumed that all floors were built using the Cobiax flat slab system. For the calculation of the fundamental frequency no further loadings besides the self-weight were implied. The material qualities were chosen for a C30/37 concrete with a Young's modulus of 28 300 N/mm² and a density of 25 kN/m³. Depending on the geometry of each system, the ball size diameter was 22.5cm or 31.5cm. For these two Cobiax flat slab systems specific qualities such as stiffness reduction and deal load were considered according to Appendix C. The boundary conditions included simply supported one-way as well as two-way spanning floors, and pin supported 1x1 and 1x2 bay slabs. The only value which changed during this comparison was the Poisson ratio. In general all examples used a Poisson ratio of 0.2, but because two of the tables used for the estimation of pin supported slabs imply a Poisson ratio of 0.3 these examples had to be modified and the Poisson ratio was increased.
Furthermore, it is important that every example used only suitable estimations for its boundary condition. A comparison of specific methods to provide appropriate solutions for different boundary conditions was waived on this occasion.

Example 1:



General properties:

Concrete: C30/37

Young's modulus E : 28,300 N/mm²

Poisson's ratio ν : 0.2

Density γ: 25 kN/m³

Thickness *h*: 30 cm

Cobiax slab properties:

Ball size Ø : 22.5cm

Factor of stiffness reduction: 0.89

Dead load reduction: 2.39 kN/m²

Equivalent beam method:

$$f_0 = \frac{\pi}{2 \cdot a^2}\sqrt{\frac{E \cdot I}{m}} = \frac{\pi}{2 \cdot 8.0^2\,\mathrm{m}^2}\sqrt{\frac{28{,}300 \cdot 10^6\ \mathrm{N}/\mathrm{m}^2\ (0.3^3 \cdot 1.0)\mathrm{m}^4 \cdot 0.89}{12 \cdot 521\ \mathrm{kg}/\mathrm{m}}} = 8.09\ \mathrm{Hz}$$

Static deflection method:

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{g}{\delta_{stat}}} = \frac{1}{2\pi}\sqrt{\frac{9.81\ \mathrm{m}/\mathrm{s}^2}{0.0051\ \mathrm{m}}} = 6.98\ \mathrm{Hz}$$

Modified static deflection method:

$$f_0 = \frac{1.277}{2\pi}\sqrt{\frac{g}{\delta_{stat}}} = \frac{1.277}{2\pi}\sqrt{\frac{9.81\ \mathrm{m}/\mathrm{s}^2}{0.0051\ \mathrm{m}}} = 8.91\ \mathrm{Hz}$$

Approximation by Jänich:

$$f_0 = \frac{\pi}{2}\sqrt{\frac{D\,g\,K}{h\,\gamma\,N}} = \frac{\pi}{2}\sqrt{\frac{59.03\cdot 10^6\,\mathrm{Nm}\cdot 9.81\,\mathrm{m}/\mathrm{s}^2\;1.22\cdot 10^{-4}\;1/\mathrm{m}^4}{0.3\mathrm{m}\cdot 17{,}033\,\mathrm{N}/\mathrm{m}^3\cdot 0.5}} = 8.26\,\mathrm{Hz}$$

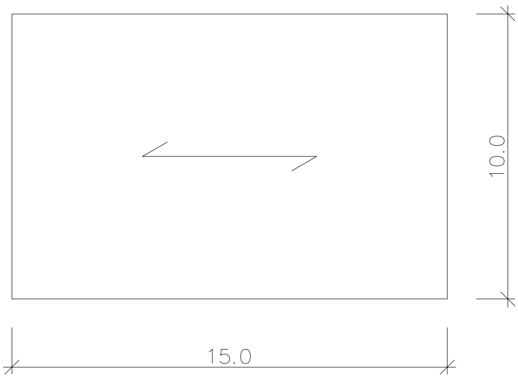
Approximation by Blevins:

$$f_0 = \frac{\lambda^2}{2\pi\,\mathrm{a}^2}\sqrt{\frac{E\,h^3}{12\,m\,(1-\nu^2)}} = \frac{9.631}{2\pi\,8.0^2\,\mathrm{m}^2}\sqrt{\frac{28{,}300\cdot 10^6\,\mathrm{N}/\mathrm{m}^2\cdot 0.3^3\,\mathrm{m}^3\cdot 0.89}{12\cdot 521\,\mathrm{kg}/\mathrm{m}^2\,\left(1-0.2^2\right)}} = 8.06\,\mathrm{Hz}$$

Comparison:

	Equivalent beam method	Static deflection method	Modified static deflection method	Approximation by Jänich	Approximation by Blevins	FEM
calculated value	8.09 Hz	6.98 Hz	8.91 Hz	8.26 Hz	8.06 Hz	8.157 Hz
Ratio %: hand calculation / FEM	99.2 %	85.6 %	109.2 %	101.3 %	98.8 %	---

Example 2:



General properties:

Concrete: C30/37

Young's modulus E : 28,300 N/mm²

Poisson's ratio ν : 0.2

Density γ: 25 kN/m³

Thickness *h*: 40 cm

Cobiax slab properties:

Ball size Ø : 31.5cm

Factor of stiffness reduction: 0.88

Dead load reduction: 3.34 kN/m²

Equivalent beam method:

$$f_0 = \frac{\pi}{2 \cdot a^2}\sqrt{\frac{E \cdot I}{m}} = \frac{\pi}{2 \cdot 15.0^2\,\mathrm{m}^2}\sqrt{\frac{28{,}300 \cdot 10^6\,\mathrm{N/m^2}\,(0.3^3 \cdot 1.0)\mathrm{m}^4 \cdot 0.88}{12 \cdot 679\,\mathrm{kg/m}}} = 3.09\,\mathrm{Hz}$$

Static deflection method:

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{g}{\delta_{stat}}} = \frac{1}{2\pi}\sqrt{\frac{9.81\,\mathrm{m/s^2}}{0.034\,\mathrm{m}}} = 2.70\,\mathrm{Hz}$$

Modified static deflection method:

$$f_0 = \frac{1.277}{2\pi}\sqrt{\frac{g}{\delta_{stat}}} = \frac{1.277}{2\pi}\sqrt{\frac{9.81\,\mathrm{m/s^2}}{0.034\,\mathrm{m}}} = 3.45\,\mathrm{Hz}$$

Approximation by Jänich:

$$f_0 = \frac{\pi}{2}\sqrt{\frac{D\,g\,K}{h\,\gamma\,N}} = \frac{\pi}{2}\sqrt{\frac{138.36\cdot 10^6\,\mathrm{Nm}\cdot 9.81\,\mathrm{m}/\mathrm{s}^2\cdot 9.88\cdot 10^{-6}\,1/\mathrm{m}^4}{0.4\mathrm{m}\cdot 16{,}650\,\mathrm{N}/\mathrm{m}^3\cdot 0.5}} = 3.15\ \mathrm{Hz}$$

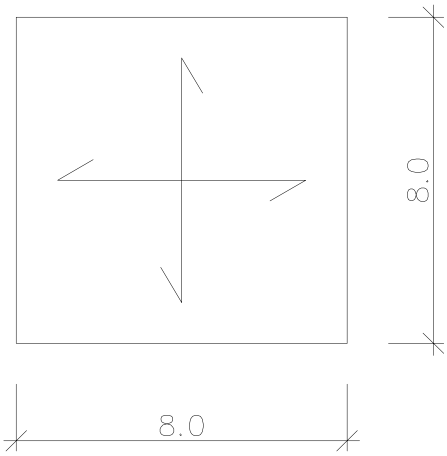
Approximation by Blevins:

$$f_0 = \frac{\lambda^2}{2\pi\,\mathrm{a}^2}\sqrt{\frac{E\,h^3}{12\,m\,(1-\nu^2)}} = \frac{9.558}{2\pi\,15.0^2\,\mathrm{m}^2}\sqrt{\frac{28{,}300\cdot 10^6\,\mathrm{N}/\mathrm{m}^2\cdot 0.4^3\,\mathrm{m}^3\cdot 0.88}{12\cdot 679\,\mathrm{kg}/\mathrm{m}^2\,\left(1-0.2^2\right)}} = 2.99\ \mathrm{Hz}$$

Comparison:

	Equivalent beam method	Static deflection method	Modified static deflection method	Approximation by Jänich	Approximation by Blevins	FEM
calculated value	3.09 Hz	2.70 Hz	3.45 Hz	3.15 Hz	2.99 Hz	3.106 Hz
Ratio %: hand calculation / FEM	99.5 %	86.9 %	111.1 %	101.4 %	96.3 %	---

Example 3:



General properties:

Concrete: C30/37

Young's modulus E : 28,300 N/mm²

Poisson's ratio ν : 0.2

Density γ: 25 kN/m³

Thickness *h*: 30 cm

Cobiax slab properties:

Ball size Ø : 22.5cm

Factor of stiffness reduction: 0.89

Dead load reduction: 2.39 kN/m²

Equivalent plate approach:

$$f_0 = \frac{\pi}{2}\sqrt{\frac{D}{m}}\left(\frac{1}{a^2}+\frac{1}{b^2}\right) = \frac{\pi}{2}\sqrt{\frac{59.03\cdot 10^6\,\mathrm{Nm}}{521\,\mathrm{kg}/\mathrm{m}^2}}\left(\frac{1}{8.0^2\,\mathrm{m}^2}+\frac{1}{8.0^2\,\mathrm{m}^2}\right) = 16.52\,\mathrm{Hz}$$

Concrete Society method:

$$\lambda_x = \frac{n_x\, a}{b}\left(\frac{EI_y}{EI_x}\right)^{1/4} = \frac{1\cdot 8.0\mathrm{m}}{8.0\mathrm{m}}\left(\frac{59.03\cdot 10^6\,\mathrm{Nm}}{59.03\cdot 10^6\,\mathrm{Nm}}\right) = 1.0$$

$$k_x = 1+\frac{1}{\lambda_x^2} = 1+\frac{1}{1.0^2} = 2.0$$

$$f_x^{'} = f_0 = k_x\,\frac{\pi}{2}\sqrt{\frac{EI_y}{mb^4}} = 2.0\,\frac{\pi}{2}\sqrt{\frac{59.03\cdot 10^6\,\mathrm{Nm}}{521\,\mathrm{kg}/\mathrm{m}^2\cdot 8.0^4\,\mathrm{m}^4}} = 16.52\,\mathrm{Hz}$$

Static deflection method:

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{g}{\delta_{stat}}} = \frac{1}{2\pi}\sqrt{\frac{9.81\ \mathrm{m}/\mathrm{s}^2}{0.00144\ \mathrm{m}}} = 13.14\ \mathrm{Hz}$$

Modified static deflection method:

$$f_0 = \frac{1.277}{2\pi}\sqrt{\frac{g}{\delta_{stat}}} = \frac{1.277}{2\pi}\sqrt{\frac{9.81\ \mathrm{m}/\mathrm{s}^2}{0.00144\ \mathrm{m}}} = 16.78\ \mathrm{Hz}$$

Approximation by Jänich:

$$f_0 = \frac{\pi}{2}\sqrt{\frac{D\,g\,K}{h\,\gamma\,N}} = \frac{\pi}{2}\sqrt{\frac{59.03\cdot 10^6\ \mathrm{Nm}\cdot 9.81\ \mathrm{m}/\mathrm{s}^2\cdot 2.44\cdot 10^{-4}\ 1/\mathrm{m}^4}{0.3\mathrm{m}\cdot 17{,}033\ \mathrm{N}/\mathrm{m}^3\cdot 0.25}} = 16.52\ \mathrm{Hz}$$

Approximation by Hearmon:

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{\frac{A^4 D_1}{a^4} + \frac{B^4 D_2}{b^4} + \frac{2CD_3}{a^2 b^2}}{m}}$$

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{\frac{\pi^4\, 59.03\cdot 10^6\ \mathrm{Nm}}{8.0^4\ \mathrm{m}^4} + \frac{\pi^4\, 59.03\cdot 10^6\ \mathrm{Nm}}{8.0^4\ \mathrm{m}^4} + \frac{2\pi^4\, 59.03\cdot 10^6\ \mathrm{Nm}}{8.0^2\ \mathrm{m}^2\cdot 8.0^2\ \mathrm{m}^2}}{521\ \mathrm{kg}/\mathrm{m}^2}} = 16.52\ \mathrm{Hz}$$

Approximation by Bachmann:

$$f_0 = \frac{\varphi_n}{a^2}\sqrt{\frac{E\,h^3}{12\,m\,(1-\nu^2)}} = \frac{3.14}{8.0^2\ \mathrm{m}^2}\sqrt{\frac{28{,}300\cdot 10^6\ \mathrm{N}/\mathrm{m}^2\cdot 0.3^3\ \mathrm{m}^3\cdot 0.89}{12\cdot 521\ \mathrm{kg}/\mathrm{m}^2\left(1-0.2^2\right)}} = 16.51\ \mathrm{Hz}$$

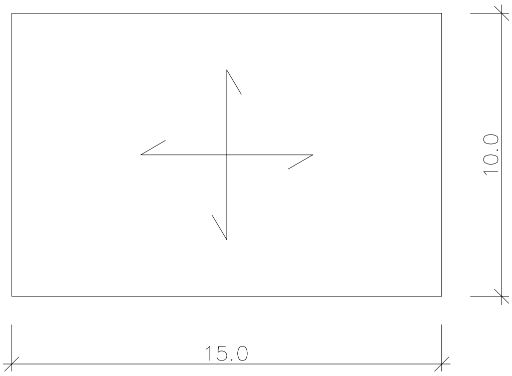
Approximation by Blevins:

$$f_0 = \frac{\lambda^2}{2\pi\, a^2}\sqrt{\frac{E\, h^3}{12\, m\,(1-\nu^2)}} = \frac{19.74}{2\pi\, 8.0^2\, m^2}\sqrt{\frac{28{,}300\cdot 10^6\, N/m^2\cdot 0.3^3\, m^3\cdot 0.89}{12\cdot 521\, kg/m^2\,\left(1-0.2^2\right)}} = 16.52\ Hz$$

Comparison:

	Equivalent plate approach	Concrete Society method	Static deflection method	Modified static deflection method	Approximation by Hearmon	Approximation by Jänich	Approximation by Bachmann	Approximation by Blevins	FEM
calculated value	16.52 Hz	16.52 Hz	13.14 Hz	16.78 Hz	16.52 Hz	16.52 Hz	16.51 Hz	16.52 Hz	16.467 Hz
Ratio %: $^{\text{hand calculation}} / _{\text{FEM}}$	100.3 %	100.3 %	79.8 %	101.9 %	100.3 %	100.3 %	100.3 %	100.3 %	---

Example 4:



General properties:

Concrete: C30/37

Young's modulus E : 28,300 N/mm²

Poisson's ratio ν : 0.2

Density γ: 25 kN/m³

Thickness *h*: 40 cm

Cobiax slab properties:

Ball size Ø : 31.5cm

Factor of stiffness reduction: 0.88

Dead load reduction: 3.34 kN/m²

Equivalent plate approach:

$$f_0 = \frac{\pi}{2}\sqrt{\frac{D}{m}}\left(\frac{1}{a^2}+\frac{1}{b^2}\right) = \frac{\pi}{2}\sqrt{\frac{138.36\cdot 10^6\,\mathrm{Nm}}{679\,\mathrm{kg}/\mathrm{m}^2}}\left(\frac{1}{15.0^2\,\mathrm{m}^2}+\frac{1}{10.0^2\,\mathrm{m}^2}\right) = 10.24\,\mathrm{Hz}$$

Concrete Society method:

$$\lambda_x = \frac{n_x\,a}{b}\left(\frac{EI_y}{EI_x}\right)^{1/4} = \frac{1\cdot 15.0\mathrm{m}}{10.0\mathrm{m}}\left(\frac{138.36\cdot 10^6\,\mathrm{Nm}}{138.36\cdot 10^6\,\mathrm{Nm}}\right) = 1.5$$

$$k_x = 1+\frac{1}{\lambda_x^2} = 1+\frac{1}{1.5^2} = 1.44$$

$$f_x^{'} = f_0 = k_x\,\frac{\pi}{2}\sqrt{\frac{EI_y}{mb^4}} = 1.44\,\frac{\pi}{2}\sqrt{\frac{132.82\cdot 10^6\,\mathrm{Nm}}{679\,\mathrm{kg}/\mathrm{m}^2\cdot 10.0^4\,\mathrm{m}^4}} = 10.00\,\mathrm{Hz}$$

Static deflection method:

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{g}{\delta_{stat}}} = \frac{1}{2\pi}\sqrt{\frac{9.81\ \mathrm{m}/\mathrm{s}^2}{0.00367\ \mathrm{m}}} = 8.23\ \mathrm{Hz}$$

Modified static deflection method:

$$f_0 = \frac{1.277}{2\pi}\sqrt{\frac{g}{\delta_{stat}}} = \frac{1.277}{2\pi}\sqrt{\frac{9.81\ \mathrm{m}/\mathrm{s}^2}{0.00367\ \mathrm{m}}} = 10.51\ \mathrm{Hz}$$

Approximation by Jänich:

$$f_0 = \frac{\pi}{2}\sqrt{\frac{D\,g\,K}{h\,\gamma\,N}} = \frac{\pi}{2}\sqrt{\frac{138.36\cdot10^6\ \mathrm{Nm}\cdot 9.81\ \mathrm{m}/\mathrm{s}^2\cdot 5.22\cdot10^{-5}\ 1/\mathrm{m}^4}{0.4\mathrm{m}\cdot 16{,}650\ \mathrm{N}/\mathrm{m}^3\cdot 0.25}} = 10.24\ \mathrm{Hz}$$

Approximation by Hearmon:

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{\frac{A^4 D_1}{a^4}+\frac{B^4 D_2}{b^4}+\frac{2CD_3}{a^2 b^2}}{m}}$$

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{\frac{\pi^4\,138.36\cdot10^6\ \mathrm{Nm}}{15.0^4\ \mathrm{m}^4}+\frac{\pi^4\,138.36\cdot10^6\ \mathrm{Nm}}{10.0^4\ \mathrm{m}^4}+\frac{2\pi^4\,138.36\cdot10^6\ \mathrm{Nm}}{15.0^2\ \mathrm{m}^2\cdot10.0^2\ \mathrm{m}^2}}{679\ \mathrm{kg}/\mathrm{m}^2}} = 10.24\ \mathrm{Hz}$$

Approximation by Bachmann:

$$f_0 = \frac{\varphi_n}{a^2}\sqrt{\frac{E\,h^3}{12\,m\,(1-\nu^2)}} = \frac{5.10}{15.0^2\ \mathrm{m}^2}\sqrt{\frac{28{,}300\cdot10^6\ \mathrm{N}/\mathrm{m}^2\cdot 0.4^3\ \mathrm{m}^3\cdot 0.88}{12\cdot 679\ \mathrm{kg}/\mathrm{m}^2\left(1-0.2^2\right)}} = 10.23\ \mathrm{Hz}$$

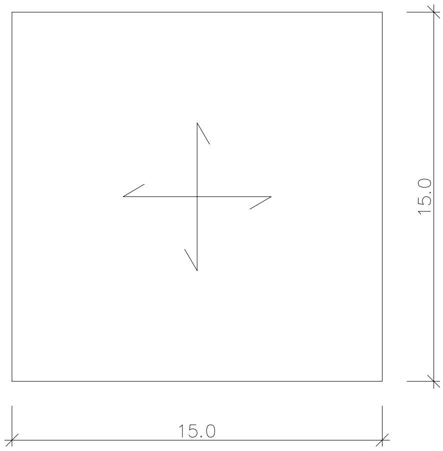
Approximation by Blevins:

$$f_0 = \frac{\lambda^2}{2\pi\, a^2}\sqrt{\frac{E\, h^3}{12\, m\,(1-\nu^2)}} = \frac{32.08}{2\pi\, 15.0^2\, m^2}\sqrt{\frac{28{,}300\cdot 10^6\, {}^{N}\!/\!_{m^2}\cdot 0.4^3\ m^3\cdot 0.88}{12\cdot 679\, {}^{kg}\!/\!_{m^2}\,\left(1-0.2^2\right)}} = 10.24\ \text{Hz}$$

Comparison:

	Equivalent plate approach	Concrete Society method	Static deflection method	Modified static deflection method	Approximation by Hearmon	Approximation by Jänich	Approximation by Bachmann	Approximation by Blevins	FEM
calculated value	10.24 Hz	10.00 Hz	8.23 Hz	10.51 Hz	10.24 Hz	10.24 Hz	10.23 Hz	10.24 Hz	10.322 Hz
Ratio %: $^{\text{hand calculation}} / _{\text{FEM}}$	99.2 %	96.9 %	79.7 %	101.8 %	99.2 %	99.2 %	99.1 %	99.2 %	---

Example 5:



General properties:

Concrete: C30/37

Young's modulus E : 28,300 N/mm²

Poisson's ratio ν : 0.2

Density γ: 25 kN/m³

Thickness *h*: 30 cm

Cobiax slab properties:

Ball size Ø : 22.5cm

Factor of stiffness reduction: 0.89

Dead load reduction: 2.39 kN/m²

Equivalent plate approach:

$$f_0 = \frac{\pi}{2}\sqrt{\frac{D}{m}}\left(\frac{1}{a^2}+\frac{1}{b^2}\right) = \frac{\pi}{2}\sqrt{\frac{138.36\cdot 10^6\,\mathrm{Nm}}{679\,\mathrm{kg}/\mathrm{m}^2}}\left(\frac{1}{15.0^2\,\mathrm{m}^2}+\frac{1}{15.0^2\,\mathrm{m}^2}\right) = 6.30\,\mathrm{Hz}$$

Concrete Society method:

$$\lambda_x = \frac{n_x\,a}{b}\left(\frac{EI_y}{EI_x}\right)^{1/4} = \frac{1\cdot 15.0\mathrm{m}}{15.0\mathrm{m}}\left(\frac{138.36\cdot 10^6\,\mathrm{Nm}}{138.36\cdot 10^6\,\mathrm{Nm}}\right) = 1.0$$

$$k_x = 1+\frac{1}{\lambda_x^2} = 1+\frac{1}{1.0^2} = 2.0$$

$$f_x^{'} = f_0 = k_x\,\frac{\pi}{2}\sqrt{\frac{EI_y}{mb^4}} = 2.0\,\frac{\pi}{2}\sqrt{\frac{132.82\cdot 10^6\,\mathrm{Nm}}{679\,\mathrm{kg}/\mathrm{m}^2\cdot 15.0^4\,\mathrm{m}^4}} = 6.18\,\mathrm{Hz}$$

Static deflection method:

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{g}{\delta_{stat}}} = \frac{1}{2\pi}\sqrt{\frac{9.81\,{}^{\mathrm{m}}\!/\!{}_{\mathrm{s}^2}}{0.00975\ \mathrm{m}}} = 5.05\ \mathrm{Hz}$$

Modified static deflection method:

$$f_0 = \frac{1.277}{2\pi}\sqrt{\frac{g}{\delta_{stat}}} = \frac{1.277}{2\pi}\sqrt{\frac{9.81\,{}^{\mathrm{m}}\!/\!{}_{\mathrm{s}^2}}{0.009.75\ \mathrm{m}}} = 6.44\ \mathrm{Hz}$$

Approximation by Jänich:

$$f_0 = \frac{\pi}{2}\sqrt{\frac{D\,g\,K}{h\,\gamma\,N}} = \frac{\pi}{2}\sqrt{\frac{138.36\cdot 10^6\ \mathrm{Nm}\cdot 9.81\,{}^{\mathrm{m}}\!/\!{}_{\mathrm{s}^2}\cdot 1.975\cdot 10^{-5}\,{}^{1}\!/\!{}_{\mathrm{m}^4}}{0.4\mathrm{m}\cdot 16{,}650\,{}^{\mathrm{N}}\!/\!{}_{\mathrm{m}^3}\cdot 0.25}} = 6.30\ \mathrm{Hz}$$

Approximation by Hearmon:

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{\frac{A^4 D_1}{a^4} + \frac{B^4 D_2}{b^4} + \frac{2CD_3}{a^2 b^2}}{m}}$$

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{\frac{\pi^4\, 138.36\cdot 10^6\ \mathrm{Nm}}{15.0^4\ \mathrm{m}^4} + \frac{\pi^4\, 138.36\cdot 10^6\ \mathrm{Nm}}{15.0^4\ \mathrm{m}^4} + \frac{2\pi^4\, 138.36\cdot 10^6\ \mathrm{Nm}}{15.0^2\ \mathrm{m}^2\cdot 15.0^2\ \mathrm{m}^2}}{679\,{}^{\mathrm{kg}}\!/\!{}_{\mathrm{m}^2}}} = 6.30\ \mathrm{Hz}$$

Approximation by Bachmann:

$$f_0 = \frac{\varphi_n}{a^2}\sqrt{\frac{E\,h^3}{12\,m\,(1-\nu^2)}} = \frac{3.14}{15.0^2\ \mathrm{m}^2}\sqrt{\frac{28{,}300\cdot 10^6\,{}^{\mathrm{N}}\!/\!{}_{\mathrm{m}^2}\cdot 0.4^3\ \mathrm{m}^3\cdot 0.88}{12\cdot 679\,{}^{\mathrm{kg}}\!/\!{}_{\mathrm{m}^2}\left(1-0.2^2\right)}} = 6.29\ \mathrm{Hz}$$

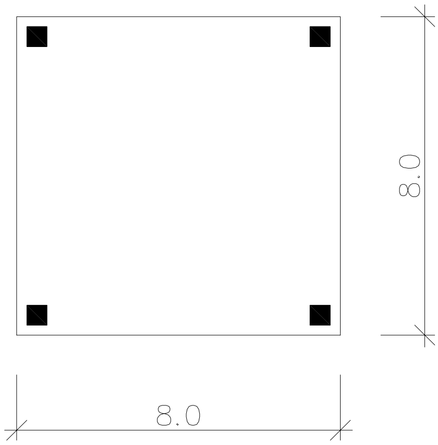
Approximation by Blevins:

$$f_0 = \frac{\lambda^2}{2\pi\, \mathrm{a}^2} \sqrt{\frac{E\, h^3}{12\, m\,(1 - \nu^2)}} = \frac{19.74}{2\pi\, 15.0^2\, \mathrm{m}^2} \sqrt{\frac{28{,}300 \cdot 10^6\, \mathrm{N}/\mathrm{m}^2 \cdot 0.4^3\ \mathrm{m}^3 \cdot 0.88}{12 \cdot 679\, \mathrm{kg}/\mathrm{m}^2\, \left(1 - 0.2^2\right)}} = 6.30\ \mathrm{Hz}$$

Comparison:

	Equivalent plate approach	Concrete Society method	Static deflection method	Modified static deflection method	Approximation by Hearmon	Approximation by Jänich	Approximation by Bachmann	Approximation by Blevins	FEM
calculated value	6.30 Hz	6.18 Hz	5.05 Hz	6.44 Hz	6.30 Hz	6.30 Hz	6.29 Hz	6.30 Hz	6.354 Hz
Ratio %: hand calculation / FEM	99.2 %	97.3 %	79.5 %	101.4 %	99.2 %	99.2 %	99.0 %	99.2 %	---

Example 6:



General properties:

Concrete: C30/37

Young's modulus E : 28,300 N/mm²

Poisson's ratio ν : 0.3

Density γ: 25 kN/m³

Thickness *h*: 30 cm

Cobiax slab properties:

Ball size Ø : 22.5cm

Factor of stiffness reduction: 0.89

Dead load reduction: 2.39 kN/m²

Static deflection method:

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{g}{\delta_S}} = \frac{1}{2\pi}\sqrt{\frac{9.81\,\mathrm{m}/\mathrm{s}^2}{0.00865\mathrm{m}}} = 5.36\,\mathrm{Hz}$$

Modified static deflection method:

$$f_0 = \frac{1.277}{2\pi}\sqrt{\frac{g}{\delta_S}} = \frac{1.277}{2\pi}\sqrt{\frac{9.81\,\mathrm{m}/\mathrm{s}^2}{0.00865\mathrm{m}}} = 6.84\,\mathrm{Hz}$$

Estimation for pin supported plates:

$$f_0 = \frac{\lambda\cdot\pi}{2a^2}\left[\sqrt{\frac{\gamma\cdot h}{D\cdot g}}\right]^{-1} = \frac{0.721\cdot\pi}{2\cdot 8.0^2\,\mathrm{m}^2}\left[\sqrt{\frac{17{,}033\,\mathrm{N}/\mathrm{m}^3\cdot 0.3\,\mathrm{m}}{62.28\cdot 10^6\ \mathrm{Nm}\cdot 9.81\,\mathrm{m}/\mathrm{s}^2}}\right]^{-1} = 6.12\,\mathrm{Hz}$$

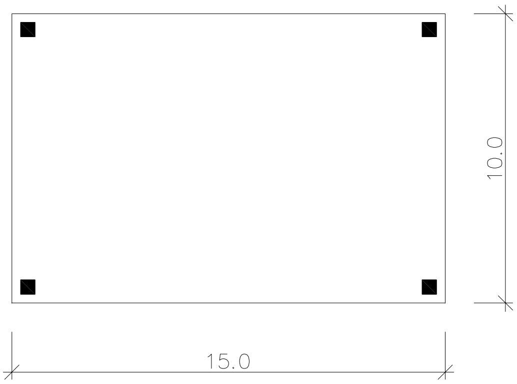
Approximation by Blevins:

$$f_0 = \frac{\lambda^2}{2\pi\, a^2}\sqrt{\frac{E\, h^3}{12\, m\,(1-\nu^2)}} = \frac{7.12}{2\pi\, 8.0^2\,\mathrm{m}^2}\sqrt{\frac{28{,}300\cdot 10^6\ \mathrm{N}/\mathrm{m}^2 \cdot 0.2^3\,\mathrm{m}^3 \cdot 0.89}{12\cdot 512\ \mathrm{kg}/\mathrm{m}^2\,\left(1-0.3^2\right)}} = 6.12\ \mathrm{Hz}$$

Comparison:

	Static deflection method	Modified static deflection method	Estimation for pin supported plates	Approximation by Blevins	FEM
calculated value	5.36 Hz	6.84 Hz	6.12 Hz	6.12 Hz	6.102 Hz
Ratio %: $^{\text{hand calculation}}/_{\text{FEM}}$	87.8 %	112.1 %	100.3 %	100.3 %	---

Example 7:



General properties:

Concrete: C30/37

Young's modulus E : 28,300 N/mm²

Poisson's ratio ν : 0.2

Density γ: 25 kN/m³

Thickness *h*: 40 cm

Cobiax slab properties:

Ball size Ø : 31.5cm

Factor of stiffness reduction: 0.88

Dead load reduction: 3.34 kN/m²

Static deflection method:

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{g}{\delta_S}} = \frac{1}{2\pi}\sqrt{\frac{9.81\,\mathrm{m/s^2}}{0.03687\mathrm{m}}} = 2.60\,\mathrm{Hz}$$

Modified static deflection method:

$$f_0 = \frac{1.277}{2\pi}\sqrt{\frac{g}{\delta_S}} = \frac{1.277}{2\pi}\sqrt{\frac{9.81\,\mathrm{m/s^2}}{0.03687\mathrm{m}}} = 3.32\,\mathrm{Hz}$$

Estimation for pin supported plates:

$$f_0 = \frac{\lambda \cdot \pi}{2a^2}\left[\sqrt{\frac{\gamma \cdot h}{D \cdot g}}\right]^{-1} = \frac{0.933 \cdot \pi}{2 \cdot 15.0^2\,\mathrm{m}^2}\left[\sqrt{\frac{16{,}650\,\mathrm{N/m^3} \cdot 0.4\,\mathrm{m}}{145.96 \cdot 10^6\,\mathrm{Nm} \cdot 9.81\,\mathrm{m/s^2}}}\right]^{-1} = 3.02\,\mathrm{Hz}$$

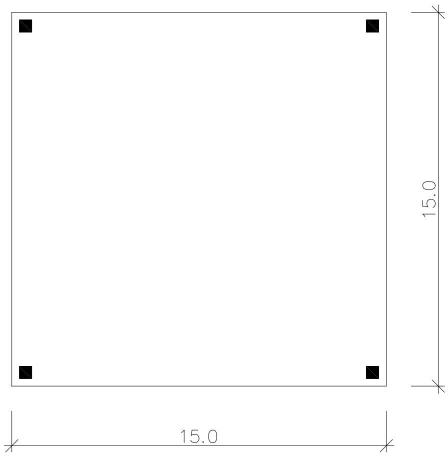
Approximation by Blevins:

$$f_0 = \frac{\lambda^2}{2\pi\, a^2}\sqrt{\frac{E\, h^3}{12\, m\,(1-\nu^2)}} = \frac{8.92}{2\pi\, 15.0^2\,\mathrm{m}^2}\sqrt{\frac{28{,}300\cdot 10^6\ \mathrm{N}/\mathrm{m}^2 \cdot 0.4^3\,\mathrm{m}^3 \cdot 0.88}{12\cdot 679\ \mathrm{kg}/\mathrm{m}^2\,\left(1-0.3^2\right)}} \;=\; 2.93\,\mathrm{Hz}$$

Comparison:

	Static deflection method	Modified static deflection method	Estimation for pin supported plates	Approximation by Blevins	FEM
calculated value	2.60 Hz	3.32 Hz	3.02 Hz	2.93 Hz	2.930 Hz
Ratio %: hand calculation / FEM	88.7 %	113.3 %	103.1 %	100.0 %	---

Example 8:



General properties:

Concrete: C30/37

Young's modulus E : 28,300 N/mm²

Poisson's ratio ν : 0.3

Density γ: 25 kN/m³

Thickness h: 40 cm

Cobiax slab properties:

Ball size Ø : 31.5cm

Factor of stiffness reduction: 0.88

Dead load reduction: 3.34 kN/m²

Static deflection method:

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{g}{\delta_S}} = \frac{1}{2\pi}\sqrt{\frac{9.81\,\mathrm{m}/\mathrm{s}^2}{0.05942\mathrm{m}}} = 2.04\,\mathrm{Hz}$$

Modified static deflection method:

$$f_0 = \frac{1.277}{2\pi}\sqrt{\frac{g}{\delta_S}} = \frac{1.277}{2\pi}\sqrt{\frac{9.81\,\mathrm{m}/\mathrm{s}^2}{0.05942\mathrm{m}}} = 2.61\,\mathrm{Hz}$$

Estimation for pin supported plates:

$$f_0 = \frac{\lambda\cdot\pi}{2a^2}\left[\sqrt{\frac{\gamma\cdot h}{D\cdot g}}\right]^{-1} = \frac{0.721\cdot\pi}{2\cdot 15.0^2\,\mathrm{m}^2}\left[\sqrt{\frac{16{,}648\,\mathrm{N}/\mathrm{m}^3\cdot 0.4\,\mathrm{m}}{145.96\cdot 10^6\,\mathrm{Nm}\cdot 9.81\,\mathrm{m}/\mathrm{s}^2}}\right]^{-1} = 2.33\,\mathrm{Hz}$$

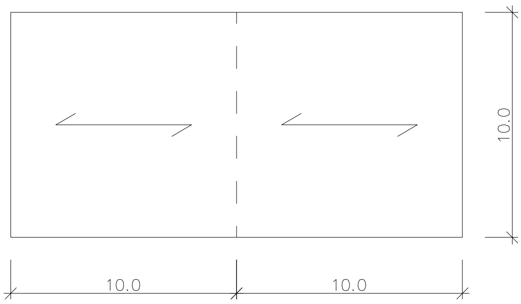
Approximation by Blevins:

$$f_0 = \frac{\lambda^2}{2\pi\, a^2}\sqrt{\frac{E\, h^3}{12\, m\,(1-\nu^2)}} = \frac{7.12}{2\pi\, 15.0^2\,\mathrm{m}^2}\sqrt{\frac{28{,}300\cdot 10^6\ \mathrm{N}/\mathrm{m}^2\cdot 0.4^3\,\mathrm{m}^3\cdot 0.88}{12\cdot 679\ \mathrm{kg}/\mathrm{m}^2\ (1-0.3^2)}} = 2.34\ \mathrm{Hz}$$

Comparison:

	Static deflection method	Modified static deflection method	Estimation for pin supported plates	Approximation by Blevins	FEM
calculated value	2.04 Hz	2.61 Hz	2.33 Hz	2.34 Hz	2.331 Hz
Ratio %: hand calculation / FEM	87.5 %	112.0 %	100.0 %	100.4 %	---

Example 9:



General properties:

Concrete: C30/37

Young's modulus E : 28,300 N/mm²

Poisson's ratio ν : 0.2

Density γ: 25 kN/m³

Thickness *h*: 30 cm

Cobiax slab properties:

Ball size Ø : 22.5cm

Factor of stiffness reduction: 0.89

Dead load reduction: 2.39 kN/m²

Approximation by Bachmann:

$$f_0 = \frac{\lambda_n}{2\pi}\left[\sqrt{\frac{\mu}{EI}}\right]^{-1} = \frac{0.1}{2\pi}\left[\sqrt{\frac{521\,\mathrm{kg}/\mathrm{m}^2}{28{,}300\cdot 10^6\,\mathrm{N}/\mathrm{m}^2\cdot 0.002\mathrm{m}^3}}\right]^{-1} = 5.25\mathrm{Hz}$$

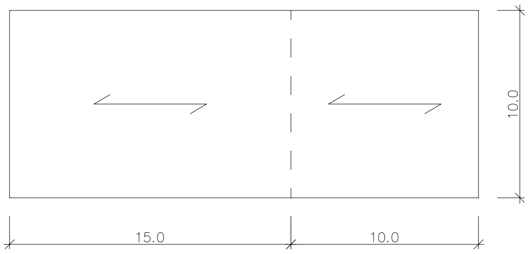
Approximation by Jänich:

$$f_0 = \frac{\pi}{2}\sqrt{\frac{D\,g\,(\sum a_i^2 K_i)}{h\,\gamma\,(\sum a_i^2 N_i)}} = \frac{\pi}{2}\sqrt{\frac{59.03\cdot 10^6\,\mathrm{Nm}\cdot 9.81\,\mathrm{m}/\mathrm{s}^2\cdot 0.01\,1/\mathrm{m}^4}{0.3\mathrm{m}\cdot 17{,}033\,\mathrm{N}/\mathrm{m}^3\cdot 100.0}} = 5.29\ \mathrm{Hz}$$

Comparison:

	Approximation by Jänich	Approximation by Bachmann	FEM
calculated value	5.29 Hz	5.25 Hz	5.211 Hz
Ratio %: hand calculation / FEM	101.5 %	100.7 %	---

Example 10:



General properties:

Concrete: C30/37

Young's modulus E : 28,300 N/mm²

Poisson's ratio ν : 0.2

Density γ: 25 kN/m³

Thickness *h*: 40 cm

Cobiax slab properties:

Ball size Ø : 31.5cm

Factor of stiffness reduction: 0.88

Dead load reduction: 3.34 kN/m²

Approximation by Bachmann:

$$f_0 = \frac{\lambda_n}{2\pi}\left[\sqrt{\frac{\mu}{EI}}\right]^{-1} = \frac{0.055}{2\pi}\left[\sqrt{\frac{679\,{}^{\mathrm{kg}}\!/\!{}_{\mathrm{m}^2}}{28{,}300\cdot 10^6\,{}^{\mathrm{N}}\!/\!{}_{\mathrm{m}^2}\cdot 0.0047\mathrm{m}^3}}\right]^{-1} = 3.87\ \mathrm{Hz}$$

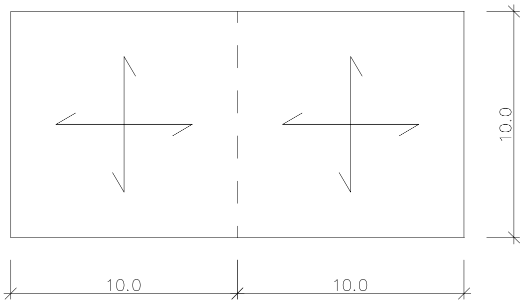
Approximation by Jänich:

$$f_0 = \frac{\pi}{2}\sqrt{\frac{D\,g\,(\sum a_i^2 K_i)}{h\,\gamma\,(\sum a_i^2 N_i)}} = \frac{\pi}{2}\sqrt{\frac{138.36\cdot 10^6\,\mathrm{Nm}\cdot 9.81\,{}^{\mathrm{m}}\!/\!{}_{\mathrm{s}^2}\cdot 0.00722\,{}^{1}\!/\!{}_{\mathrm{m}^2}}{0.4\,\mathrm{m}\cdot 16{,}650\,{}^{\mathrm{N}}\!/\!{}_{\mathrm{m}^3}\cdot 162.5.0\,\mathrm{m}^2}} = 4.73\ \mathrm{Hz}$$

Comparison:

	Approximation by Jänich	Approximation by Bachmann	FEM
calculated value	4.73 Hz	3.87 Hz	3.785 Hz
Ratio %: $^{\text{hand calculation}} / _{\text{FEM}}$	125.0 %	102.2 %	---

Example 11:



General properties:

Concrete: C30/37

Young's modulus E : 28,300 N/mm²

Poisson's ratio ν : 0.2

Density γ: 25 kN/m³

Thickness *h*: 30 cm

Cobiax slab properties:

Ball size Ø : 22.5cm

Factor of stiffness reduction: 0.89

Dead load reduction: 2.39 kN/m²

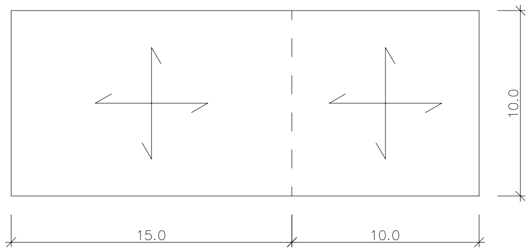
Approximation by Jänich:

$$f_0 = \frac{\pi}{2}\sqrt{\frac{D\,g\,(\sum a_i^2 K_i)}{h\,\gamma\,(\sum a_i^2 N_i)}} = \frac{\pi}{2}\sqrt{\frac{59.03\cdot 10^6\,\mathrm{Nm}\cdot 9.81\,\mathrm{m/s^2}\cdot 0.02\,\mathrm{1/m^2}}{0.3\,\mathrm{m}\cdot 17{,}033\,\mathrm{N/m^3}\cdot 50.0\,\mathrm{m^2}}} = 10.58\ \mathrm{Hz}$$

Comparison:

	Approximation by Jänich	FEM
calculated value	11.60 Hz	10.599 Hz
Ratio %: $^{\text{hand calculation}}/_{\text{FEM}}$	109.4 %	---

Example 12:



General properties:

Concrete: C30/37

Young's modulus E : 28,300 N/mm²

Poisson's ratio ν : 0.2

Density γ: 25 kN/m³

Thickness *h*: 40 cm

Cobiax slab properties:

Ball size Ø : 31.5cm

Factor of stiffness reduction: 0.88

Dead load reduction: 3.34 kN/m²

Approximation by Jänich:

$$f_0 = \frac{\pi}{2}\sqrt{\frac{D\,g\,K}{h\,\gamma\,N}} = \frac{\pi}{2}\sqrt{\frac{138.36\cdot 10^6\,\mathrm{Nm}\cdot 9.81\,\mathrm{m}/\mathrm{s}^2\cdot 0.0217\;1/\mathrm{m}^2}{0.4\,\mathrm{m}\cdot 16{,}650\,\mathrm{N}/\mathrm{m}^3\cdot 81.25\,\mathrm{m}^2}} = 11.60\ \mathrm{Hz}$$

Comparison:

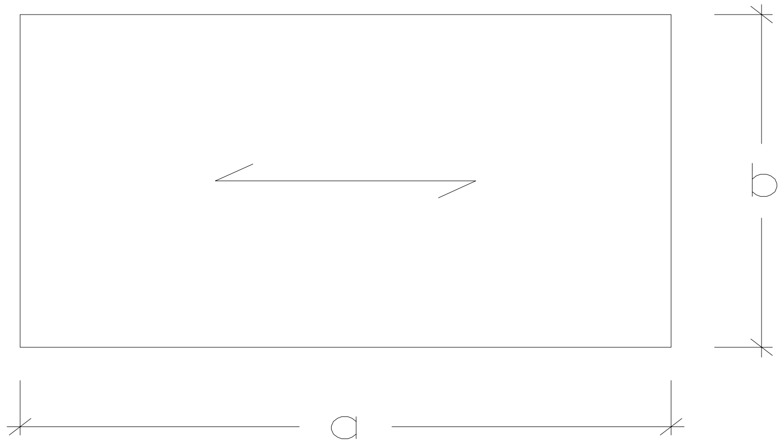
	Approximation by Jänich	FEM
calculated value	10.58 Hz	10.515 Hz
Ratio %: hand calculation / FEM	100.6 %	---

		Equivalent beam method	Equivalent plate approach	Concrete Society method	Static deflection method	Modified static deflection method	Approximation by Hearmon	Approximation by Jänich	Estimation for pin supported plates	Approximation by Bachmann	Approximation by Blevins	FEM
Example 1	calculated values f_0 [Hz]	8.09	---	---	6.98	8.91	---	8.26	---	---	8.06	8.157
	Ratio %: $^{\text{hand calculation}}/_{\text{FEM}}$	99.2 %			85.6 %	109.2 %		101.3 %			98.8 %	
Example 2	calculated values f_0 [Hz]	3.09	---	---	2.7	3.45	---	3.15	---	---	2.99	3.106
	Ratio %: $^{\text{hand calculation}}/_{\text{FEM}}$	99.5 %			86.9 %	111.1 %		101.4 %			96.3 %	
Example 3	calculated values f_0 [Hz]	---	16.52	16.52	13.14	16.78	16.52	16.52	---	16.51	16.52	16.467
	Ratio %: $^{\text{hand calculation}}/_{\text{FEM}}$		100.3 %	100.3 %	79.8 %	101.9 %	100.3 %	100.3 %		100.3 %	100.3 %	
Example 4	calculated values f_0 [Hz]	---	10.24	10.00	8.23	10.51	10.24	10.24	---	10.23	10.24	10.322
	Ratio %: $^{\text{hand calculation}}/_{\text{FEM}}$		99.2 %	96.9 %	79.7 %	101.8 %	99.2 %	99.2 %		99.1 %	99.2 %	
Example 5	calculated values f_0 [Hz]	---	6.30	6.18	5.05	6.44	6.30	6.30	---	6.29	6.30	6.354
	Ratio %: $^{\text{hand calculation}}/_{\text{FEM}}$		99.2 %	97.3 %	79.5 %	101.4 %	99.2 %	99.2 %		99.0 %	99.2 %	
Example 6	calculated values f_0 [Hz]	---	---	---	5.36	6.84	---	---	6.12	---	6.12	6.102
	Ratio %: $^{\text{hand calculation}}/_{\text{FEM}}$				87.8 %	112.1 %			100.3 %		100.3 %	
Example 7	calculated values f_0 [Hz]	---	---	---	2.60	3.32	---	---	3.02	---	2.93	2.93
	Ratio %: $^{\text{hand calculation}}/_{\text{FEM}}$				88.7 %	113.3 %			103.1 %		100.0 %	
Example 8	calculated values f_0 [Hz]	---	---	---	2.04	2.61	---	---	2.33	---	2.34	2.331
	Ratio %: $^{\text{hand calculation}}/_{\text{FEM}}$				87.5 %	112.0 %			100.0 %		100.4 %	
Example 9	calculated values f_0 [Hz]	---	---	---	---	---	---	5.29	---	5.25	---	5.211
	Ratio %: $^{\text{hand calculation}}/_{\text{FEM}}$							101.5 %		100.7 %		
Example 10	calculated values f_0 [Hz]	---	---	---	---	---	---	4.73	---	3.87	---	3.785
	Ratio %: $^{\text{hand calculation}}/_{\text{FEM}}$							125.0 %		102.2 %		
Example 11	calculated values f_0 [Hz]	---	---	---	---	---	---	10.58	---	---	---	10.515
	Ratio %: $^{\text{hand calculation}}/_{\text{FEM}}$							100.6 %				
Example 12	calculated values f_0 [Hz]	---	---	---	---	---	---	11.6	---	---	---	10.599
	Ratio %: $^{\text{hand calculation}}/_{\text{FEM}}$							109.4 %				
arithmetic mean in %		99.3 %	99.6 %	98.2 %	84.4 %	107.8 %	99.6 %	104.2 %	101.1 %	100.3 %	99.3 %	
standard deviation		0.153	0.540	1.540	3.797	4.887	0.540	7.893	1.395	0.998	1.284	

Table A.1 Summary of approximate hand calculation

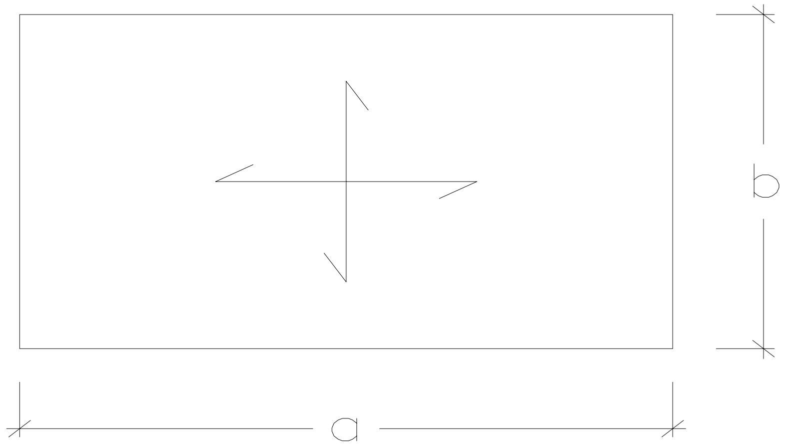
APPENDIX B

(Calculated Values by FEM)



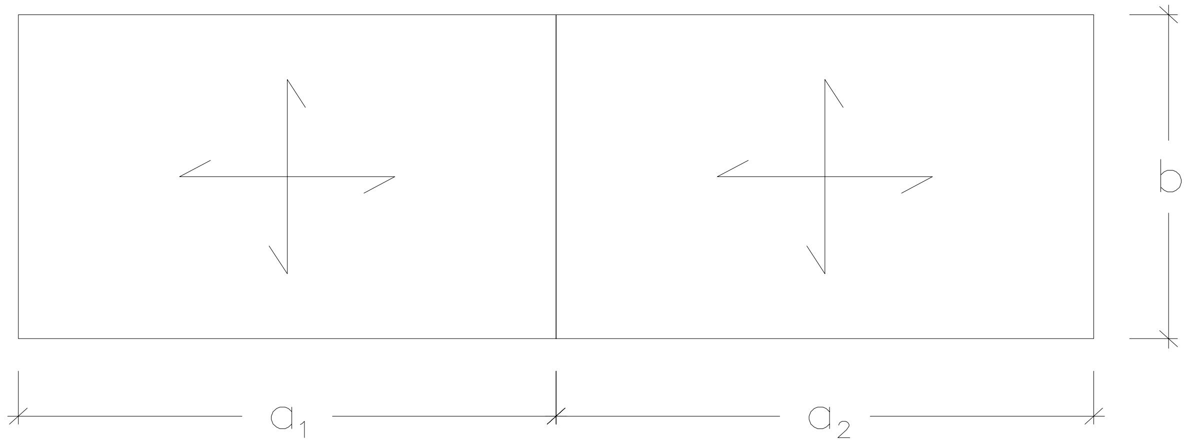
h	sphere Ø	a	b		q = 0 kN/m²			q = 1.25 kN/m²			q = 2.50kN/m²			q = 3.75 kN/m²			q = 5.00 kN/m²		
					f_0 [Hz]			f_0 [Hz]			f_0 [Hz]			f_0 [Hz]			f_0 [Hz]		
[cm]	[cm]	[m]	[m]		1.Mode	2. Mode	3. Mode	1.Mode	2. Mode	3. Mode	1.Mode	2. Mode	3. Mode	1.Mode	2. Mode	3. Mode	1.Mode	2. Mode	3. Mode
30	22.5	6	4	cs	12.709	29.716	51.071	11.655	27.252	46.836	10.826	25.315	43.506	10.153	23.739	40.798	9.591	22.425	38.540
				ss	11.548	27.001	46.406	10.821	25.301	43.484	10.216	23.887	41.053	9.702	22.686	38.989	9.259	21.649	37.208
30	22.5	8	6	cs	7.153	15.202	28.719	6.560	13.942	26.338	6.094	12.950	24.465	5.714	12.144	22.942	5.398	11.472	21.673
				ss	6.500	13.813	26.096	6.091	12.944	24.453	5.750	12.220	23.086	5.461	11.606	21.925	5.212	11.075	20.923
30	22.5	8	8	cs	7.172	12.343	27.587	6.578	11.319	25.299	6.110	10.514	23.500	5.730	9.860	22.038	5.412	9.314	20.818
				ss	6.517	11.215	25.067	6.107	10.509	23.488	5.765	9.921	22.175	5.476	9.422	21.060	5.225	8.992	20.098
30	22.5	10	6	cs	4.572	11.687	18.371	4.193	10.718	16.848	3.895	9.956	15.650	3.652	9.336	14.676	3.450	8.819	13.864
				ss	4.154	10.619	16.693	3.893	9.951	15.642	3.675	9.394	14.768	3.490	8.922	14.025	3.331	8.514	13.384
30	22.5	10	8	cs	4.585	9.263	18.413	4.205	8.495	16.886	3.906	7.891	15.685	3.663	7.399	14.709	3.460	6.990	13.895
				ss	4.166	8.417	16.731	3.904	7.887	15.678	3.686	7.446	14.801	3.500	7.071	14.057	3.340	6.748	13.415
30	22.5	10	10	cs	4.596	7.902	17.671	4.215	7.247	16.206	3.915	6.732	15.054	3.671	6.313	14.117	3.468	5.963	13.335
				ss	4.176	7.181	16.057	3.913	6.728	15.046	3.694	6.352	14.205	3.508	6.033	13.491	3.348	5.757	12.874
40	31.5	12	10	cs	4.389	8.603	17.617	4.095	8.027	16.513	3.853	7.552	15.593	3.649	7.153	14.811	3.475	6.811	14.135
				ss	3.941	7.725	15.818	3.748	7.346	15.096	3.581	7.019	14.463	3.434	6.732	13.904	3.304	6.477	13.405
40	31.5	15	10	cs	2.806	6.562	11.284	2.613	6.111	10.508	2.455	5.741	9.873	2.323	5.431	9.340	2.210	5.167	8.886
				ss	2.520	5.892	10.132	2.393	5.596	9.623	2.284	5.340	9.183	2.188	5.116	8.799	2.104	4.919	8.459
40	31.5	15	12	cs	2.811	5.679	11.297	2.617	5.289	10.519	2.459	4.969	9.883	2.326	4.701	9.350	2.213	4.472	8.895
				ss	2.524	5.100	10.143	2.397	4.843	9.633	2.287	4.622	9.193	2.188	4.420	8.781	2.104	4.249	8.442
60	45	17	12	cs	3.367	7.511	13.543	3.201	7.140	12.874	3.057	6.819	12.295	2.931	6.538	11.788	2.819	6.289	11.339
				ss	3.009	6.712	12.102	2.901	6.471	11.668	2.804	6.255	11.278	2.716	6.059	10.924	2.636	5.880	10.602

Table B.1 Simply supported slab, one-way spanning



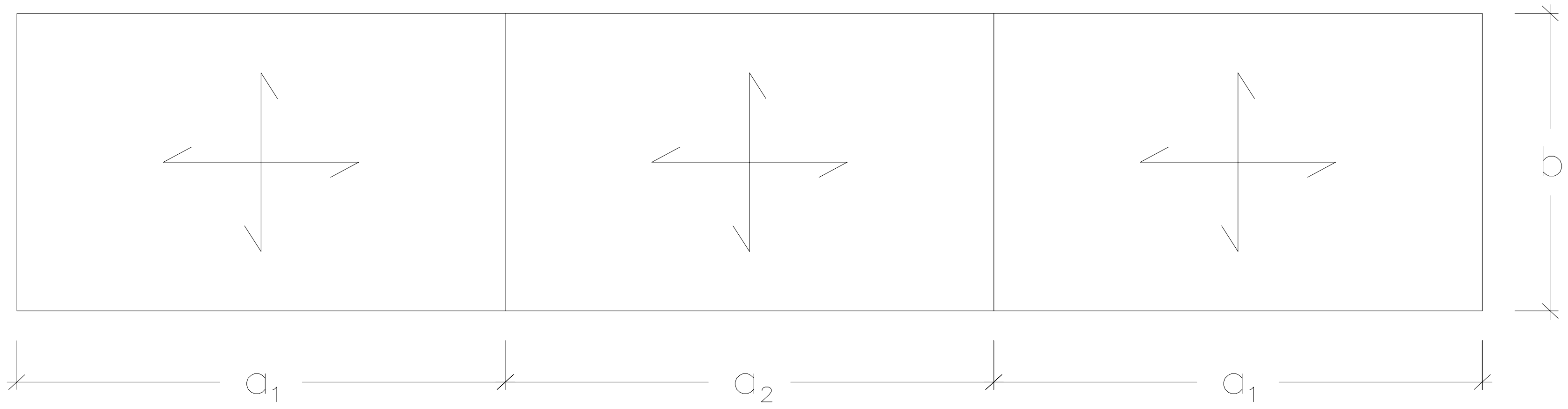
h	sphere Ø	a	b		q = 0 kN/m²			q = 1.25 kN/m²			q = 2.50kN/m²			q = 3.75 kN/m²			q = 5.00 kN/m²		
					f_0 [Hz]			f_0 [Hz]			f_0 [Hz]			f_0 [Hz]			f_0 [Hz]		
[cm]	[cm]	[m]	[m]		1.Mode	2. Mode	3. Mode	1.Mode	2. Mode	3. Mode	1.Mode	2. Mode	3. Mode	1.Mode	2. Mode	3. Mode	1.Mode	2. Mode	3. Mode
30	22.5	6	4	cs	41.741	80.241	125.960	38.280	73.587	115.516	35.558	68.355	107.302	33.345	64.100	100.623	31.499	60.552	95.053
				ss	37.928	72.910	114.453	35.540	68.320	107.248	33.553	64.500	101.251	31.866	61.257	96.160	30.410	58.458	91.767
30	22.5	6	6	cs	25.783	63.432	63.578	23.645	58.172	58.306	21.964	54.035	54.160	20.597	50.672	50.789	19.457	47.867	47.978
				ss	23.428	57.637	57.770	21.953	54.008	54.133	20.725	50.989	51.106	19.683	48.425	48.537	18.784	46.212	46.319
30	22.5	8	6	cs	20.070	41.456	58.041	18.406	38.018	53.229	17.097	35.315	49.444	16.033	33.117	46.366	15.145	31.284	43.800
				ss	18.237	37.669	52.739	17.088	35.297	49.419	16.133	33.324	4.666	15.322	31.648	44.310	14.622	30.202	42.286
30	22.5	8	8	cs	14.473	35.973	36.057	13.273	32.990	33.067	12.329	30.645	30.716	11.562	28.737	28.804	10.922	27.147	27.209
				ss	13.151	32.687	32.763	12.323	30.629	30.700	11.634	28.917	28.984	11.049	27.463	27.526	10.544	26.208	26.269
30	22.5	10	6	cs	17.497	31.200	54.208	16.046	28.613	49.713	14.905	26.578	46.178	13.977	24.924	43.304	13.204	23.545	40.907
				ss	15.899	28.350	49.256	14.898	26.565	46.155	14.065	25.080	43.574	13.358	23.819	41.383	12.747	22.731	39.493
30	22.5	10	8	cs	11.879	25.771	33.554	10.894	23.634	30.772	10.120	21.953	28.584	9.490	20.587	26.804	8.965	19.447	25.321
				ss	10.794	23.416	30.489	10.115	21.942	28.569	9.549	20.715	26.972	9.069	19.674	25.616	8.655	18.775	24.445
30	22.5	10	10	cs	9.280	23.163	23.175	8.510	21.242	21.253	7.905	19.732	19.742	7.413	18.504	18.513	7.003	17.479	17.488
				ss	8.432	21.047	21.058	7.901	19.722	19.732	7.459	18.619	18.629	7.084	17.683	17.692	6.761	16.875	16.884
40	31.5	12	10	cs	10.837	24.135	29.989	10.092	22.474	27.926	9.482	21.115	26.237	8.970	19.977	24.822	8.533	19.004	23.614
				ss	9.731	21.670	26.926	9.241	20.580	25.572	8.819	19.640	24.404	8.450	18.818	23.383	8.124	18.091	22.479
40	31.5	12	12	cs	8.883	22.185	22.192	8.272	20.659	20.666	7.771	19.410	19.416	7.352	18.363	18.369	6.994	17.469	17.475
				ss	7.976	19.920	19.926	7.575	18.918	18.924	7.229	18.054	18.060	6.926	17.298	17.304	6.658	16.630	16.635
40	31.5	15	10	cs	9.243	17.755	28.399	8.608	16.534	26.446	8.087	15.534	24.847	7.651	14.696	23.506	7.278	13.981	22.362
				ss	8.300	15.942	25.499	7.882	15.140	24.217	7.522	14.449	23.111	7.207	13.844	22.143	6.929	13.309	21.288
40	31.5	15	12	cs	7.289	15.807	20.606	6.788	14.719	19.188	6.377	13.829	18.028	6.033	13.083	17.056	5.740	12.446	16.226
				ss	6.545	14.193	18.502	6.216	13.479	17.572	5.932	12.863	16.769	5.683	12.325	16.067	5.464	11.849	15.446
60	45.0	17	15	cs	7.797	18.021	20.939	7.412	17.131	19.905	7.079	16.361	19.011	6.787	15.687	18.227	6.528	15.089	17.533
				ss	6.967	16.104	18.712	6.717	15.526	18.041	6.493	15.007	17.437	6.289	14.536	16.890	6.103	14.108	16.392
60	45.0	17	17	cs	6.825	17.053	17.060	6.488	16.211	16.218	6.197	15.482	15.489	5.941	14.844	14.850	5.715	14.279	14.285
				ss	6.099	15.239	15.245	5.881	14.693	14.699	5.684	14.201	14.207	5.506	13.756	13.762	5.343	13.350	13.356

Table B.2 Simply supported slab, two-way spanning



h	sphere Ø	a_1	a_2	b		q = 0 kN/m²			q = 1.25 kN/m²			q = 2.50kN/m²			q = 3.75 kN/m²			q = 5.00 kN/m²		
						f_0 [Hz]			f_0 [Hz]			f_0 [Hz]			f_0 [Hz]			f_0 [Hz]		
[cm]	[cm]	[m]	[m]	[m]		1.Mode	2. Mode	3. Mode	1.Mode	2. Mode	3. Mode	1.Mode	2. Mode	3. Mode	1.Mode	2. Mode	3. Mode	1.Mode	2. Mode	3. Mode
30	22.5	8	6	6	cs	20.947	28.369	44.568	19.210	26.017	40.873	17.844	24.167	37.966	16.733	22.663	35.603	15.807	21.408	33.632
					ss	19.034	25.778	40.497	17.835	24.155	37.947	16.838	22.804	35.826	15.991	21.658	34.024	15.261	20.668	32.470
30	22.5	8	8	8	cs	14.467	17.281	35.991	13.268	15.848	33.007	12.324	14.722	30.660	11.557	13.805	28.752	10.918	13.041	27.160
					ss	13.146	15.703	32.704	12.318	14.714	30.645	11.630	13.892	28.931	11.045	13.193	27.477	10.540	12.590	26.221
30	22.5	10	8	8	cs	12.368	16.079	27.495	11.342	14.746	25.215	10.536	13.697	23.422	9.880	12.845	21.964	9.333	12.134	20.748
					ss	11.238	14.610	24.983	10.531	13.690	23.410	9.942	12.925	22.101	9.442	12.275	20.990	9.010	11.714	20.031
30	22.5	10	10	10	cs	9.264	11.067	22.987	8.496	10.149	21.081	7.892	9.427	19.582	7.401	8.840	18.363	6.991	8.351	17.347
					ss	8.418	10.056	20.887	7.888	9.423	19.572	7.447	8.896	18.478	7.073	8.448	17.549	6.749	8.062	16.747
40	31.5	12	10	10	cs	11.279	14.286	25.639	10.503	13.303	23.875	9.868	12.498	22.431	9.335	11.824	21.222	8.881	11.249	20.189
					ss	10.127	12.827	23.021	9.618	12.182	21.863	9.178	11.625	20.865	8.794	11.139	19.991	8.454	10.708	19.219
40	31.5	12	12	12	cs	8.879	10.615	22.100	8.268	9.885	20.580	7.769	9.288	19.335	7.350	8.787	18.293	6.992	8.359	17.402
					ss	7.973	9.532	19.844	7.572	9.052	18.846	7.226	8.639	17.985	6.923	8.277	17.232	6.656	7.957	16.566
40	31.5	15	12	12	cs	7.594	9.860	16.847	7.072	9.182	15.688	6.644	8.627	14.740	6.286	8.161	13.945	5.980	7.764	13.266
					ss	6.819	8.853	15.127	6.476	8.408	14.366	6.180	8.024	13.710	5.921	7.688	13.136	5.693	7.391	12.629
40	31.5	15	15	15	cs	5.679	6.790	14.157	5.288	6.323	13.183	4.968	5.941	12.386	4.701	5.620	11.718	4.472	5.347	11.148
					ss	5.099	6.097	12.712	4.843	5.790	12.073	4.621	5.526	11.521	4.428	5.295	11.039	4.257	5.090	10.612
60	45	17	15	15	cs	8.074	9.921	18.984	7.675	9.431	18.047	7.330	9.007	17.236	7.028	8.635	16.525	6.761	8.307	15.896
					ss	7.215	8.865	16.965	6.956	8.547	16.357	6.724	8.261	15.809	6.513	8.002	15.314	6.321	7.766	14.862
60	45	17	17	17	cs	6.818	8.155	17.008	6.482	7.752	16.168	6.190	7.404	15.441	5.935	7.098	14.805	5.709	6.828	14.241
					ss	6.093	7.287	15.199	5.874	7.026	14.654	5.678	6.791	14.164	5.500	6.578	13.719	5.338	6.384	13.315

Table B.3 Two span slab, two-way spanning

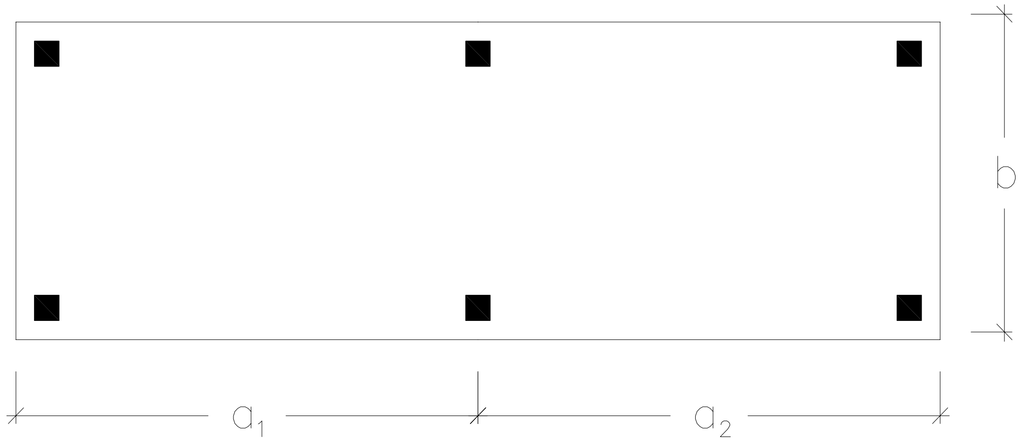


h	sphere Ø	a_1	a_2	b		q = 0 kN/m²			q = 1.25 kN/m²			q = 2.50kN/m²			q = 3.75 kN/m²			q = 5.00 kN/m²		
						f_0 [Hz]			f_0 [Hz]			f_0 [Hz]			f_0 [Hz]			f_0 [Hz]		
[cm]	[cm]	[m]	[m]	[m]		1.Mode	2. Mode	3. Mode	1.Mode	2. Mode	3. Mode	1.Mode	2. Mode	3. Mode	1.Mode	2. Mode	3. Mode	1.Mode	2. Mode	3. Mode
30	22.5	8	6	6	CS	20,798	21,373	30,869	19,073	19,601	28,309	17,717	26,296	18,207	16,614	17,074	24,659	15,695	16,129	23,294
					SS	18,898	19,420	28,049	17,708	18,198	26,283	16,718	17,180	24,814	15,878	16,316	23,566	15,152	15,571	22,489
30	22.5	8	8	8	CS	14,469	15,815	18,957	13,269	14,504	17,385	12,325	13,472	16,149	11,558	12,634	15,144	10,918	11,934	14,306
					SS	13,147	14,370	17,226	12,319	13,466	16,141	11,630	12,713	15,239	11,046	12,074	14,472	10,541	11,522	13,811
30	22.5	8	10	8	CS	12,853	15,685	16,823	11,787	14,384	15,428	10,949	13,361	14,331	10,267	12,530	13,439	9,699	11,836	12,695
					SS	11,679	14,252	15,286	10,944	13,355	14,324	10,332	12,608	13,523	9,812	11,974	12,843	9,364	11,427	12,256
30	22.5	10	8	8	CS	12,263	12,689	17,577	11,247	11,637	16,119	10,447	10,809	14,973	9,797	10,136	14,041	9,254	9,575	13,264
					SS	11,143	11,530	15,971	10,442	10,804	14,966	9,858	14,129	14,129	9,362	9,687	13,419	8,934	9,244	12,805
30	22.5	10	10	10	CS	9,257	10,120	12,164	8,490	9,280	11,156	7,886	8,621	10,362	7,395	8,084	9,717	6,986	7,637	9,180
					SS	8,412	9,195	11,053	7,882	8,616	10,357	7,441	8,135	9,778	7,067	7,726	9,287	6,744	7,373	8,862
30	22.5	10	12	10	CS	8,456	10,042	10,889	7,755	9,209	9,986	7,203	8,554	9,276	6,755	8,022	8,698	6,381	7,578	8,217
					SS	7,683	9,124	9,894	7,200	8,550	9,271	6,797	8,072	8,753	6,455	7,666	8,313	6,160	7,316	7,933
40	31.5	12	10	10	CS	11,159	11,618	15,699	10,391	10,819	14,619	9,763	10,165	13,735	9,237	9,616	12,994	8,787	9,148	12,361
					SS	10,020	10,432	14,096	9,516	9,907	13,387	9,081	9,455	12,775	8,701	9,059	12,241	8,365	8,709	11,768
40	31.5	12	12	12	CS	8,869	9,692	11,655	8,259	9,025	10,853	7,760	8,479	10,197	7,341	8,022	9,647	6,984	7,631	9,177
					SS	7,964	8,702	10,465	7,563	8,265	9,939	7,218	7,887	9,485	6,916	7,557	9,088	6,649	7,265	8,737
40	31.5	12	14	12	CS	8,243	9,637	10,551	7,676	8,974	9,825	7,212	8,432	9,231	6,823	7,977	8,733	6,491	7,589	8,308
					SS	7,401	8,653	9,474	7,029	8,218	8,997	6,708	7,843	8,586	6,427	7,514	8,227	6,179	7,224	7,909
40	31.5	15	12	12	CS	7,515	7,776	10,803	6,998	7,241	10,060	6,575	6,803	9,451	6,220	6,436	8,942	5,917	6,123	8,506
					SS	6,747	6,982	9,700	6,408	6,631	9,212	6,115	6,328	8,791	5,859	6,063	8,423	5,633	5,829	8,098
40	31.5	15	15	15	CS	5,680	6,212	7,464	5,289	5,785	6,951	4,969	5,435	6,530	4,701	5,142	6,178	4,472	4,892	5,877
					SS	5,100	5,578	6,702	4,843	5,297	6,365	4,622	5,055	6,074	4,429	4,844	5,820	4,258	4,657	5,595
60	45	17	15	15	CS	7,992	8,409	10,944	7,597	7,994	10,403	7,256	7,635	9,936	6,956	7,320	9,526	6,692	7,041	9,163
					SS	7,142	7,515	9,779	6,886	7,245	9,429	6,655	7,003	9,113	6,447	6,783	8,828	6,256	6,583	8,567
60	45	17	17	17	CS	6,820	7,456	8,959	6,483	7,088	8,517	6,191	6,769	8,134	5,936	6,490	7,799	5,710	6,243	7,502
					SS	6,094	6,663	8,006	5,876	6,424	7,719	5,679	6,209	7,461	5,501	6,014	7,227	5,339	5,837	7,014

Table B.4 Three span slab, two-way spanning

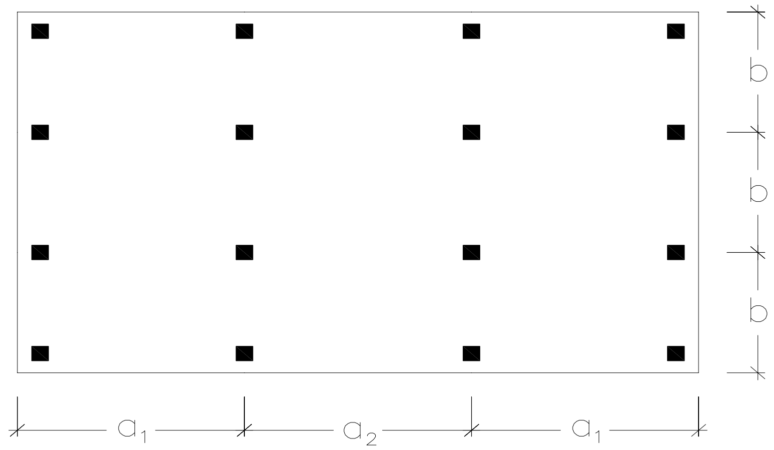
h	sphere Ø	a	b		q = 0 kN/m²			q = 1.25 kN/m²			q = 2.50kN/m²			q = 3.75 kN/m²			q = 5.00 kN/m²		
					f_0 [Hz]			f_0 [Hz]			f_0 [Hz]			f_0 [Hz]			f_0 [Hz]		
[cm]	[cm]	[m]	[m]		1.Mode	2. Mode	3. Mode	1.Mode	2. Mode	3. Mode	1.Mode	2. Mode	3. Mode	1.Mode	2. Mode	3. Mode	1.Mode	2. Mode	3. Mode
30	22.5	6	4	CS	11,741	29,544	34,245	10,767	27,094	31,406	10,002	25,168	29,172	9,379	23,601	27,357	8,860	22,295	25,842
				SS	10,669	26,845	31,117	9,997	25,155	29,158	9,438	23,749	27,528	8,963	22,555	26,144	8,554	21,524	24,949
30	22.5	6	6	CS	9,255	21,336	21,361	8,487	19,567	19,590	7,884	18,176	18,197	7,393	17,044	17,064	6,984	16,101	16,120
				SS	8,409	19,387	19,410	7,880	18,167	18,188	7,439	17,151	17,171	7,065	16,289	16,307	6,743	15,544	15,562
30	22.5	8	6	CS	6,339	15,074	16,940	5,813	13,824	15,536	5,400	12,841	14,431	5,064	12,042	13,533	4,784	11,375	12,784
				SS	5,760	13,697	15,393	5,397	12,834	14,424	5,096	12,117	13,617	4,839	11,508	12,933	4,618	10,982	12,342
30	22.5	8	8	CS	5,206	12,004	12,019	4,774	11,009	11,022	4,435	10,226	10,239	4,159	9,590	9,601	3,929	9,059	9,070
				SS	4,731	10,908	10,921	4,433	10,221	10,234	4,185	9,650	9,661	3,974	9,164	9,176	3,793	8,746	8,756
30	22.5	10	6	CS	4,327	11,644	13,618	3,968	10,678	12,489	3,686	9,919	11,601	3,457	9,302	10,879	3,265	8,787	10,276
				SS	3,932	10,580	12,374	3,684	9,914	11,595	3,478	9,360	10,947	3,303	8,889	10,396	3,152	8,483	9,921
30	22.5	10	8	CS	3,939	9,162	10,068	3,612	8,402	9,233	3,355	7,805	8,576	3,146	7,319	8,042	2,972	6,914	7,597
				SS	3,579	8,325	9,148	3,354	7,801	8,572	3,166	7,365	8,093	3,007	6,995	7,686	2,869	6,675	7,335
30	22.5	10	10	CS	3,334	7,689	7,694	3,057	7,051	7,056	2,840	6,550	6,554	2,663	6,142	6,146	2,516	5,802	5,806
				SS	3,029	6,987	6,991	2,838	6,547	6,551	2,680	6,181	6,185	2,545	5,870	5,874	2,429	5,602	5,605
40	31.5	12	10	CS	3,686	8,493	9,188	3,432	7,909	8,556	3,225	7,431	8,039	3,051	7,030	7,605	2,902	6,688	7,235
				SS	3,310	7,626	8,250	3,143	7,243	7,835	3,000	6,912	7,477	2,874	6,622	7,164	2,763	6,367	6,887
40	31.5	12	12	CS	3,190	7,360	7,362	2,971	6,854	6,856	2,791	6,440	6,441	2,641	6,092	6,094	2,512	5,796	5,797
				SS	2,865	6,609	6,610	2,721	6,276	6,278	2,596	5,990	5,991	2,488	5,739	5,740	2,392	5,517	5,519
40	31.5	15	10	CS	2,591	6,529	7,559	2,412	6,080	7,039	2,266	5,712	6,613	2,144	5,404	6,257	2,040	5,141	5,952
				SS	2,182	5,499	6,367	2,072	5,223	6,047	1,978	4,984	5,770	1,895	4,775	5,529	1,822	4,591	5,315
40	31.5	15	12	CS	2,414	5,618	6,174	2,248	5,232	5,749	2,112	4,916	5,402	1,998	4,650	5,110	1,901	4,424	4,862
				SS	2,168	5,045	5,544	2,059	4,791	5,265	1,965	4,572	5,024	1,882	4,381	4,814	1,810	4,212	4,628
40	31.5	15	15	CS	2,043	4,715	4,718	1,903	4,391	4,393	1,788	4,126	4,128	1,691	3,903	3,905	1,609	3,713	3,715
				SS	1,835	4,234	4,236	1,743	4,021	4,023	1,663	3,837	3,839	1,593	3,677	3,679	1,532	3,535	3,537
60	45.0	17	10	CS	3,193	8,714	10,191	3,036	8,284	9,687	2,899	7,911	9,252	2,780	7,585	8,871	2,674	7,296	8,533
				SS	2,854	7,787	9,107	2,751	7,508	8,780	2,659	7,257	8,486	2,576	7,029	8,220	2,500	6,822	7,978
60	45.0	17	17	CS	2,451	5,655	5,660	2,330	5,375	5,380	2,225	5,134	5,138	2,134	4,922	4,927	2,052	4,735	4,739
				SS	2,190	5,053	5,058	2,112	4,872	4,876	2,041	4,709	4,713	1,977	4,561	4,565	1,919	4,427	4,431

Table B.5 1x1 bay slab, supported by columns



h	sphere Ø	a_1	a_2	b		q = 0 kN/m²			q = 1.25 kN/m²			q = 2.50kN/m²			q = 3.75 kN/m²			q = 5.00 kN/m²		
						f_0 [Hz]			f_0 [Hz]			f_0 [Hz]			f_0 [Hz]			f_0 [Hz]		
[cm]	[cm]	[m]	[m]	[m]		1.Mode	2. Mode	3. Mode	1.Mode	2. Mode	3. Mode	1.Mode	2. Mode	3. Mode	1.Mode	2. Mode	3. Mode	1.Mode	2. Mode	3. Mode
30	22.5	8.0	6.0	6.0	cs	7,434	10,624	16,140	6,817	9,743	14,802	6,333	9,050	13,749	5,938	8,487	12,893	5,610	8,017	12,180
					ss	6,752	9,663	14,639	6,327	9,055	13,717	5,973	8,548	12,951	5,673	8,118	12,299	5,414	7,748	11,737
30	22.5	8.0	8.0	8.0	cs	5,909	6,064	12,160	5,421	5,568	11,165	5,035	5,172	10,371	4,722	4,850	9,726	4,461	4,582	9,188
					ss	5.37	5.51	11.049	5.032	5.163	10.353	4.75	4.874	9.775	4.511	4.629	9.283	4.305	4.418	8.859
30	22.5	10.0	8.0	8.0	cs	4.576	5.956	9.753	4.197	5.462	8.945	3.898	5.074	8.309	3.656	4.758	7.791	3.453	4.495	7.36
					ss	4.158	5.412	8.862	3.896	5.071	8.304	3.679	4.788	7.84	3.494	4.547	7.446	3.334	4.339	7.106
30	22.5	10.0	10.0	10.0	cs	3.773	3.877	7.779	3.46	3.556	7.134	3.214	3.303	6.627	3.014	3.098	6.214	2.847	5.876	6.125
					ss	3.429	3.523	7.068	3.213	3.301	6.623	3.033	3.117	6.253	2.881	2.96	5.939	2.825	5.667	5.91
40	31.5	12.0	10.0	10.0	cs	4.267	5.258	9.013	3.973	4.896	8.393	3.733	4.6	7.886	3.532	4.352	7.461	3.36	4.14	7.097
					ss	3.831	4.721	8.093	3.638	4.484	7.686	3.472	4.279	7.335	3.327	4.1	7.028	3.198	3.941	6.757
40	31.5	12.0	12.0	12.0	cs	3.618	3.713	7.449	3.369	3.457	6.937	3.165	3.248	6.518	2.994	3.073	6.166	2.849	2.923	5.866
					ss	3.248	3.334	6.689	3.085	3.166	6.353	2.944	3.021	6.062	2.821	2.895	5.809	2.712	2.783	5.584
40	31.5	15.0	12.0	12.0	cs	2.803	3.651	5.975	2.61	3.4	5.564	2.452	3.194	5.228	2.32	3.022	4.946	2.207	2.875	4.705
					ss	2.517	3.278	5.365	2.39	3.113	5.095	2.281	2.971	4.863	2.186	2.847	4.659	2.101	2.737	4.479
40	31.5	15.0	15.0	10.0	cs	2.694	3.483	6.539	2.508	3.243	6.089	2.357	3.047	5.721	2.23	2.883	5.412	2.121	2.742	5.149
					ss	2.313	2.379	4.774	2.154	2.216	4.445	2.024	2.082	4.177	1.915	1.969	3.951	1.821	1.874	3.759
40	31.5	15.0	15.0	15.0	cs	2.419	3.127	5.871	2.297	2.97	5.576	2.192	2.834	5.321	2.1	2.716	5.098	2.019	2.611	4.901
					ss	2.077	2.136	4.286	1.973	2.029	4.071	1.882	1.936	3.885	1.804	1.855	3.722	1.734	1.784	3.578
60	45.0	17.0	15.0	15.0	cs	3.148	3.603	6.579	2.993	3.425	6.254	2.858	3.271	5.973	2.74	3.136	5.727	2.636	3.016	5.509
					ss	2.813	3.219	5.879	2.712	3.104	5.669	2.622	3	5.479	2.539	2.906	5.307	2.464	2.82	5.151
60	45.0	17.0	17.0	10.0	cs	3.273	4.463	8.72	3.112	4.242	8.289	2.972	4.052	7.917	2.849	3.885	7.59	2.741	3.737	7.301
					ss	2.925	3.988	7.792	2.82	3.845	7.513	2.726	3.716	7.261	2.641	3.6	7.034	2.563	3.494	6.826
60	45.0	17.0	17.0	17.0	cs	2.776	2.855	5.729	2.639	2.714	5.446	2.521	2.592	5.201	2.417	2.485	4.987	2.325	2.391	4.797
					ss	2.482	2.551	5.121	2.393	2.46	4.937	2.313	2.378	4.772	2.241	2.303	4.623	2.174	2.235	4.486

Table B.6 2x1 bay slab, supported by columns



h	sphere Ø	a_1	a_2	b		q = 0 kN/m²			q = 1.25 kN/m²			q = 2.50kN/m²			q = 3.75 kN/m²			q = 5.00 kN/m²		
						f_0 [Hz]			f_0 [Hz]			f_0 [Hz]			f_0 [Hz]			f_0 [Hz]		
[cm]	[cm]	[m]	[m]	[m]		1.Mode	2. Mode	3. Mode	1.Mode	2. Mode	3. Mode	1.Mode	2. Mode	3. Mode	1.Mode	2. Mode	3. Mode	1.Mode	2. Mode	3. Mode
30	22.5	8	8	6	CS	7.014	8.129	8.295	6.432	7.455	7.608	5.975	6.925	7.067	5.603	6.494	6.627	5.293	6.134	6.26
					SS	6.373	7.386	7.538	5.972	6.921	7.063	5.638	6.534	6.668	5.354	6.206	6.333	5.11	5.922	6.044
30	22.5	8	8	8	CS	6.489	6.897	6.904	5.951	6.325	6.332	5.528	5.876	5.882	5.184	5.51	5.515	4.897	5.205	5.21
					SS	5.897	6.267	6.274	5.525	5.873	5.879	5.216	5.544	5.55	4.954	5.265	5.271	4.728	5.025	5.03
30	22.5	9	9	6	CS	5.583	6.734	6.773	5.12	6.175	6.211	4.756	5.736	5.77	4.46	5.379	5.41	4.213	5.081	5.111
					SS	5.073	6.119	6.154	4.753	5.733	5.767	4.487	5.413	5.444	4.262	5.141	5.171	4.067	4.906	4.934
30	22.5	9	9	9	CS	5.137	5.458	5.463	4.711	5.006	5.01	4.376	4.65	4.653	4.104	4.36	4.364	3.877	4.119	4.122
					SS	4.668	4.96	4.964	4.374	4.648	4.651	4.129	4.388	4.391	3.922	4.167	4.17	3.743	3.977	3.98
30	22.5	10	10	6	CS	4.548	5.591	5.705	4.171	5.127	5.232	3.875	4.763	4.86	3.633	4.466	4.557	3.432	4.219	4.305
					SS	4.133	5.08	5.184	3.873	4.76	4.858	3.656	4.494	4.586	3.472	4.268	4.355	3.314	4.073	4.156
30	22.5	10	10	10	CS	4.163	4.421	4.43	3.818	4.055	4.062	3.546	3.766	3.773	3.326	3.532	3.539	3.142	3.336	3.343
					SS	3.783	4.017	4.025	3.545	3.764	3.772	3.347	3.554	3.561	3.178	3.375	3.382	3.033	3.221	3.227
30	22.5	11	11	6	CS	3.77	4.677	4.923	3.457	4.289	4.515	3.211	3.984	4.194	3.012	3.736	3.933	2.845	3.529	3.715
					SS	3.425	4.249	4.473	3.21	3.982	4.192	3.03	3.759	3.957	2.878	3.57	3.758	2.747	3.407	3.587
30	22.5	11	11	11	CS	3.443	3.66	3.66	3.157	3.356	3.357	2.933	3.118	3.118	2.75	2.924	2.924	2.598	2.762	2.762
					SS	3.128	3.325	3.326	2.931	3.116	3.117	2.767	2.942	2.942	2.628	2.794	2.794	2.508	2.666	2.667
40	31.5	12	12	8	CS	4.334	5.225	5.25	4.036	4.865	4.889	3.792	4.571	4.593	3.588	4.325	4.345	3.413	4.114	4.134
					SS	3.892	4.691	4.714	3.696	4.456	4.477	3.527	4.252	4.272	3.38	4.074	4.093	3.249	3.917	3.935
40	31.5	12	12	12	CS	3.991	4.241	4.247	3.717	3.949	3.954	3.492	3.711	3.715	3.304	3.51	3.515	3.143	3.34	3.344
					SS	3.584	3.808	3.813	3.403	3.617	3.621	3.248	3.451	3.456	3.112	3.307	3.311	2.992	3.179	3.183
40	31.5	13	13	10	CS	3.651	4.189	4.294	3.4	3.9	3.999	3.195	3.665	3.757	3.022	3.467	3.554	2.875	3.298	3.381
					SS	3.279	3.761	3.856	3.114	3.572	3.662	2.971	3.409	3.494	2.847	3.266	3.348	2.737	3.14	3.219
40	31.5	13	13	13	CS	3.397	3.608	3.614	3.164	3.359	3.365	2.972	3.156	3.162	2.812	2.986	2.991	2.675	2.841	2.846
					SS	3.049	3.248	3.249	2.896	3.085	3.085	2.764	2.944	2.944	2.648	2.82	2.821	2.546	2.711	2.712
40	31.5	14	14	8	CS	3.206	3.961	4.104	2.986	3.689	3.822	2.805	3.466	3.591	2.654	3.279	3.397	2.525	3.119	3.232
					SS	2.879	3.557	3.685	2.734	3.378	3.5	2.609	3.224	3.34	2.5	3.089	3.2	2.403	2.969	3.077
40	31.5	14	14	14	CS	2.932	3.117	3.118	2.731	2.903	2.904	2.565	2.727	2.728	2.427	2.58	2.581	2.309	2.455	2.455
					SS	2.633	2.799	2.800	2.500	2.658	2.659	2.386	2.537	2.538	2.286	2.431	2.431	2.198	2.337	2.337
40	31.5	15	8	10	CS	3.249	3.47	3.677	3.025	3.231	3.424	2.842	3.036	3.217	2.689	2.872	3.044	2.558	2.732	2.895
					SS	2.917	3.116	3.302	2.77	2.959	3.136	2.644	2.824	2.992	2.533	2.706	2.867	2.435	2.601	2.756
40	31.5	15	15	10	CS	2.774	3.345	3.364	2.584	3.115	3.132	2.427	2.926	2.943	2.296	2.768	2.784	2.185	2.634	2.649
					SS	2.491	3.003	3.02	2.366	2.852	2.869	2.258	2.722	2.737	2.163	2.608	2.623	2.08	2.507	2.522
40	31.5	15	15	15	CS	2.553	2.714	2.715	2.378	2.527	2.528	2.234	2.374	2.375	2.113	2.246	2.247	2.01	2.137	2.138
					SS	2.292	2.437	2.438	2.177	2.314	2.315	2.078	2.209	2.209	1.991	2.116	2.117	1.914	2.034	2.035
60	45	16	16	12	CS	3.728	4.315	4.42	3.544	4.102	4.201	3.385	3.917	4.012	3.245	3.756	3.847	3.122	3.613	3.701
					SS	3.332	3.856	3.949	3.212	3.718	3.808	3.105	3.593	3.68	3.007	3.48	3.565	2.919	3.378	3.46
60	45	16	16	16	CS	3.463	3.681	3.683	3.292	3.499	3.501	3.144	3.342	3.344	3.014	3.204	3.206	2.9	3.082	3.084
					SS	3.095	3.289	3.292	2.984	3.171	3.174	2.884	3.065	3.067	2.793	2.969	2.971	2.711	2.882	2.883
60	45	17	12	12	CS	3.724	4.064	4.212	3.54	3.864	4.004	3.381	3.69	3.824	3.241	3.538	3.666	3.118	3.403	3.527
					SS	3.327	3.632	3.764	3.208	3.502	3.629	3.101	3.385	3.508	3.004	3.279	3.398	2.915	3.182	3.297
60	45	17	17	10	CS	3.348	4.126	4.236	3.183	3.922	4.027	3.04	3.746	3.846	2.914	3.592	3.687	2.803	3.455	3.547
					SS	2.992	3.687	3.785	2.885	3.555	3.649	2.788	3.436	3.527	2.701	3.328	3.417	2.621	3.23	3.316
60	45	17	17	17	CS	3.065	3.258	3.26	2.914	3.097	3.099	2.783	2.958	2.96	2.668	2.836	2.838	2.567	2.728	2.729
					SS	2.739	2.912	2.913	2.641	2.807	2.809	2.553	2.713	2.715	2.473	2.628	2.63	2.4	2.551	2.552

Table B.7 3x3 bay slab, supported by columns

APPENDIX C

(Cobiax Information)

project	span dimension [m]	h [cm]	Ø [cm]
Zollvereinschool Essen (D)	max. 17 m	50 / 52	36
Hessischer Landtag Wiesbaden (D)	max. 17 m	40 / 45 / 50	18 / 31,5 / 36
Mainova Frankfurt (D)	max. 10,6 m	23 / 25 / 30 / 35 / 39 / 40	18 / 22,5 / 27 / 31,5
Shopping Mall Palladium Praga (CZ)	8,40 x 8,40 m	24 / 40	18 / 27
Novartis Basel (CH)	6 m x 10 m	35	22.5
Newcastle College, Newcastle Tyne & Wear (UK)	7.5 / 8.5 / 7.5 m x 5.5	30 / 32,5	18
Residential Ubiale Bergamo (IT)	10 m	60	45
Parking BRG Freistadt (AT)	16,00 x 5,00 m	55 / 62	45
Spedition Gebrüder Weiss Maria Lanzendorf (AT)	8,00 m – in one direction	30	18
SF Swiss Television Zürich (CH)	9,6 m x 8 m	30	22.5
Peugeot Center Moosseedorf (CH)	15 m x 10 m	40 / 45	31,5 / 36
Wylerpark Bern (CH)	10,4 m x 9,8 m	30	22.5
Eclipse Park, Maidstone Kent (UK)	6 x 6 m	30	18
Sheffield University LCR Sheffield (UK)	9 x 9 m	34	27
Iprona Lana (IT)	9,60 x 7,25 m	60 / 40	31,5 / 45
Commercial Centre Settevalli Perugia (IT)	12,50 m	45 / 47	36

Table C.1 Existing Cobiax projects

Steifigkeitsfaktoren zur Berücksichtigung der Verminderung durch Hohlkörper

Zur Berücksichtigung der Verminderung der Steifigkeit infolge der eingebauten Hohlkörper werden nachfolgend Steifigkeitsfaktoren für die Hohlkörperdecke angegeben. Die Werte beruhen auf Berechnungen für den Zustand I bei zentrischer Kugellage. Die Einflüsse für den Zustand II wurden anhand von Biegeversuchen überprüft. Gemäß der Auswertung dieser Versuche ist die Abminderung infolge Zustand I maßgebend. Mit diesen Faktoren kann eine Verformungsberechnung der Decken durchgeführt werden, wobei die reduzierte Eigenlast zu berücksichtigen ist.

Deckenstärke h_{cb} [cm]	23 *	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Hohlkörper D_{cb} [cm]	**18**																	
Verhältniswert h_{cb}/D_{cb} [-]	1,28	1,33	1,39	1,44	1,5	1,56	1,61	1,67	1,72	1,78	1,83	1,89	1,94	2	2,06	2,11	2,17	2,22
Verhältniswert I_{cobiax}/I_{massiv} [-]	0,88	0,89	0,90	0,92	0,92	0,93	0,94	0,94	0,95	0,95	0,96	0,96	0,97	0,97	0,97	0,97	0,97	0,98

Deckenstärke h_{cb} [cm]	28 *	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
Hohlkörper D_{cb} [cm]	**22,5**																	
Verhältniswert h_{cb}/D_{cb} [-]	1,24	1,29	1,33	1,38	1,42	1,47	1,51	1,56	1,6	1,64	1,69	1,73	1,78	1,82	1,87	1,91	1,96	2
Verhältniswert I_{cobiax}/I_{massiv} [-]	0,87	0,88	0,89	0,90	0,91	0,92	0,93	0,93	0,94	0,94	0,95	0,95	0,95	0,96	0,96	0,96	0,97	0,97

Deckenstärke h_{cb} [cm]	34 *	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51
Hohlkörper D_{cb} [cm]	**27**																	
Verhältniswert h_{cb}/D_{cb} [-]	1,26	1,3	1,33	1,37	1,41	1,44	1,48	1,52	1,56	1,59	1,63	1,67	1,7	1,74	1,78	1,81	1,85	1,89
Verhältniswert I_{cobiax}/I_{massiv} [-]	0,87	0,88	0,89	0,90	0,91	0,92	0,92	0,93	0,93	0,94	0,94	0,94	0,95	0,95	0,95	0,96	0,96	0,96

Deckenstärke h_{cb} [cm]	40 *	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57
Hohlkörper D_{cb} [cm]	**31,5**																	
Verhältniswert h_{cb}/D_{cb} [-]	1,27	1,3	1,33	1,37	1,4	1,43	1,46	1,49	1,52	1,56	1,59	1,62	1,65	1,68	1,71	1,75	1,78	1,81
Verhältniswert I_{cobiax}/I_{massiv} [-]	0,88	0,88	0,89	0,90	0,91	0,91	0,92	0,92	0,93	0,93	0,94	0,94	0,94	0,95	0,95	0,95	0,95	0,96

Deckenstärke h_{cb} [cm]	45 *	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62
Hohlkörper D_{cb} [cm]	**36**																	
Verhältniswert h_{cb}/D_{cb} [-]	1,25	1,28	1,31	1,33	1,36	1,39	1,42	1,44	1,47	1,5	1,53	1,56	1,58	1,61	1,64	1,67	1,69	1,72
Verhältniswert I_{cobiax}/I_{massiv} [-]	0,87	0,88	0,89	0,89	0,90	0,90	0,91	0,92	0,92	0,92	0,93	0,93	0,94	0,94	0,94	0,94	0,95	0,95

Deckenstärke h_{cb} [cm]	52 *	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69
Hohlkörper D_{cb} [cm]	**40,5**																	
Verhältniswert h_{cb}/D_{cb} [-]	1,28	1,31	1,33	1,36	1,38	1,41	1,43	1,46	1,48	1,51	1,53	1,56	1,58	1,6	1,63	1,65	1,68	1,7
Verhältniswert I_{cobiax}/I_{massiv} [-]	0,88	0,89	0,89	0,90	0,90	0,91	0,91	0,92	0,92	0,93	0,93	0,93	0,94	0,94	0,94	0,94	0,95	0,95

Deckenstärke h_{cb} [cm]	58 *	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75
Hohlkörper D_{cb} [cm]	**45**																	
Verhältniswert h_{cb}/D_{cb} [-]	1,29	1,31	1,33	1,36	1,38	1,4	1,42	1,44	1,47	1,49	1,51	1,53	1,56	1,58	1,6	1,62	1,64	1,67
Verhältniswert I_{cobiax}/I_{massiv} [-]	0,88	0,89	0,89	0,90	0,90	0,91	0,91	0,92	0,92	0,92	0,93	0,93	0,93	0,94	0,94	0,94	0,94	0,94

* empfohlene Mindestdeckenstärke   Wert bei exzentrischer Hohlkörperlage

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Zweiachsige Hohlkörperdecke

Verformungsberechnung
Steifigkeitsfaktor

Anlage 6

zur allgemeinen
Bauaufsichtlichen Zulassung

vom xx.xx.xxxx

Table C.2 Factors considering reduced stiffness

Sphere diameter	[cm]	18.00	22.50	27.00	31.50	36.00	40.50	45.00
Min. centre distance	[cm]	20.00	25.00	30.00	35.00	40.00	45.00	50.00
Max. amount of spheres	[1/m²]	25.00	16.00	11.11	8.16	6.25	4.94	4.00
Recommended deck thickness	[cm]	23.00	28.00	34.00	40.00	45.00	52.00	58.00
Dead load reduction per sphere	[kN]	0.08	0.15	0.26	0.41	0.61	0.87	1.19
Max. dead load reduction	[kN/m²]	1.91	2.39	2.86	3.34	3.82	4.29	4.77
Stiffness factor	[-]	0.88	0.87	0.87	0.88	0.87	0.88	0.88
Shear factor	[-]	0.55	0.55	0.55	0.55	0.55	0.55	0.55

Table C.3 Cobiax parameters

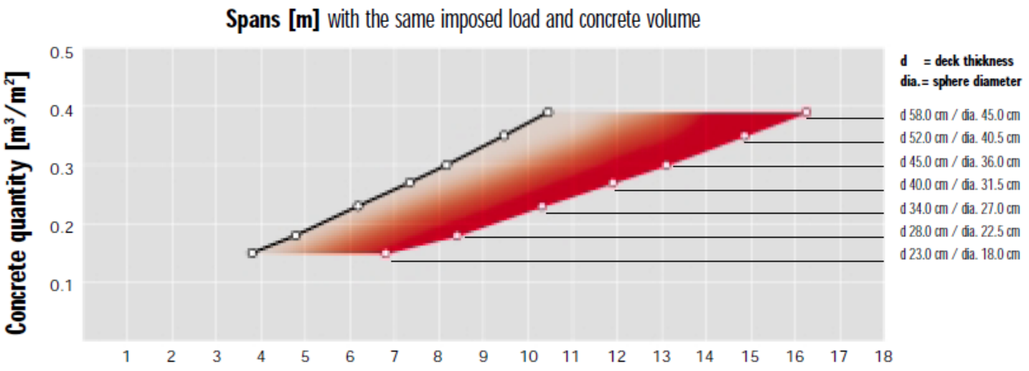

Figure C.1 Comparison of spans and concrete quantity

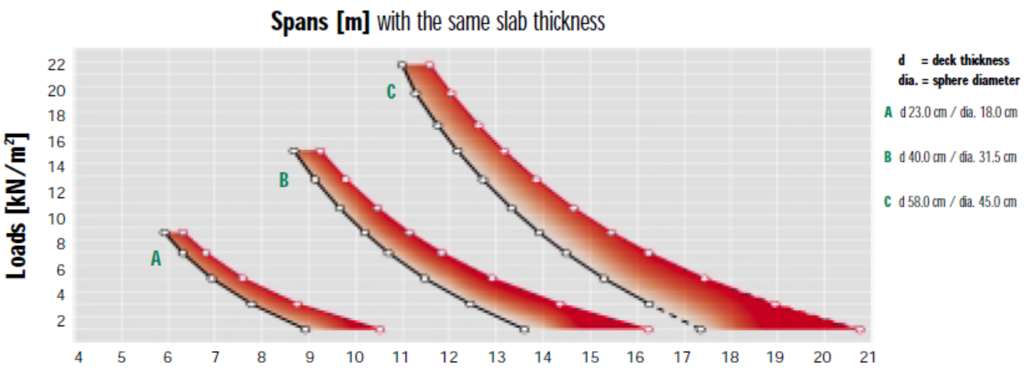

Figure C.2 Comparison of spans and loads