Natural frequency of Cobiax® flat slabs

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ABSTRACT

The continuing trend towards large open floors, free of partitions, and increased slenderness of concrete slabs increases the likelihood of human discomfort due to floor vibrations induced by human activities. This paper covers several areas concerned with addressing this problem. Some recommendations of acceptance limits provided by different literature and the involved problems are presented. Afterwards the accuracy of simplified hand calculation methods for the approximate estimation of concrete slabs' fundamental frequencies is investigated. Furthermore the vibration behaviour of a specific biaxial hollow concrete slab, the Cobiax flat slab, is investigated. By using finite element software its natural frequency is obtained, leading to a comparison to conventional solid slabs and an evaluation of the vibration performance of Cobiax flat slabs.

Keywords: Cobiax flat slab; Dynamic properties; Floors; Natural frequency; Vibration

INTRODUCTION

In the past, the importance of considering floor vibration increased as result from growing numbers of complaints in residential buildings and offices. Generally, floor vibration gives people an 'unpleasant' feeling and prompts fear of structural collapse which leads to an impairment of people's quality of life and working conditions. The prevention of these problems should be considered during the design stage. Besides machinery-induced vibration which can be usually avoided by additional isolation between source and floor, human-induced vibration is the main area of consideration. Case studies, like presented by Hanagan [6] show the problem of perceptible floor motion occurred due to human activities as walking or jumping. Because humans act as source and receiver of vibrations it is not possible to isolate them like machinery. Thus, the structure itself must be considered and modified to prevent annoying floor vibration. One way of addressing this problem is to increase the natural frequency to a level which can hardly be perceived by a building's occupants.

ACCEPTANCE CRITERIA

Evaluation of measured or calculated values of floor vibration is a complex subject. Because human perception to floor vibration varies greatly and depends on many interrelated factor like for example,

- Direction of motion
- Personal characteristics
- Timing and duration
- Current activity

accurate limit values can not be provided. However, since the pioneering work of Reiher and Meister [8] many investigations were carried out to improve the knowledge of human perception limits.

Nowadays, most national codes provide acceptance criteria which should be followed. The British Standard BS 6472 [2] includes different graphs with acceptance limits dependent on natural frequency and acceleration as shown in figure 1. It considers all three possible directions in which vibration may occur as well as variation in places and time.

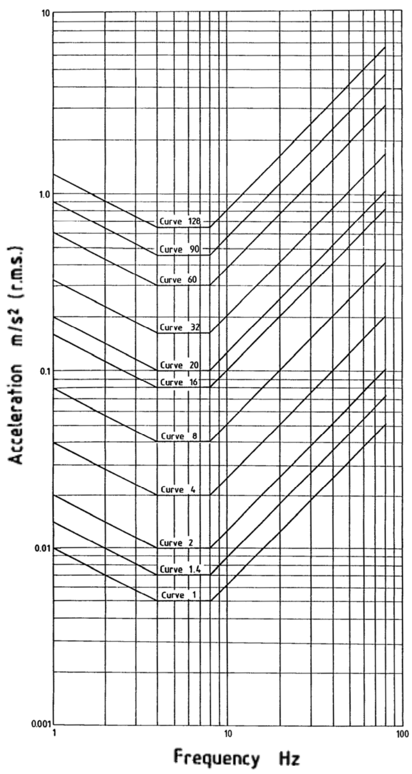

Figure 1 BS 6472 (1992) Building vibration z-axis curves for acceleration (r.m.s.)

The German Standard DIN 4150-2 [5] uses special formula to calculate the perception parameter *KB*. It relates on different vibration qualities and must be limited to specified values. Similar to the British Standard these limit values depends on occupancy and time of day.

In addition to national codes many investigations and case studies produced independent and individual recommendations. They are provided in form of graphs, tables or formulas and applicable to any type of floor.

SIMPLIFIED HAND CALCULATION METHODS

Besides the use of finite element software, which is taken for granted nowadays many simplified hand calculation methods exist. Different literature provides tables and equations to calculate the fundamental frequency of a concrete slab. Due to the fact that a slab is continuum structure, it has infinite numbers of natural frequencies. However, this may be neglected because of the main importance of the fundamental frequency which is the lowest and therefore presumably the first excited frequency.

Several approximations were included to obtain simple equations easy to apply. For example they assume an elastic behaviour of concrete slabs which leads to an application of simple linear dynamics theory. The elastic behaviour also neglected changes in stiffness and therefore changes in natural frequency after cracking. Furthermore these approximations are usually applicable for truly fixed or simply supported slabs and do not consider any restraints in-between. Obviously, it may not be expected to obtain 100% accurate solutions by applying these methods.

For this purpose the accuracy of fundamental frequencies obtained by simplified hand calculation methods was investigated.

Ten different approaches for calculation the fundamental frequency provided by existing literature were applied to common floor designs. To consider the variety of different set-ups, 12 examples with changing dimensions and boundary conditions were regarded. They included one-way as well as two-way spanning floors, single span and continuous floors and simply supported and pin supported floors.

The fundamental frequency of each floor was calculated by proper simplifications developed for the examples' specific boundary condition. In addition all floors were modelled with finite element software and their fundamental frequency was calculated. This led to a comparison between frequencies obtained by both ways and an evaluation of hand-calculated values for natural frequencies.

Figure 2 shows the comparison between fundamental frequencies f_0 obtained by hand calculation and finite element software. It is noticeable that besides the static deflection and the modified static deflection method presented by Blenvis [3] as well as two values calculated with an approximation presented by Jänich [7] all frequencies are very accurate.

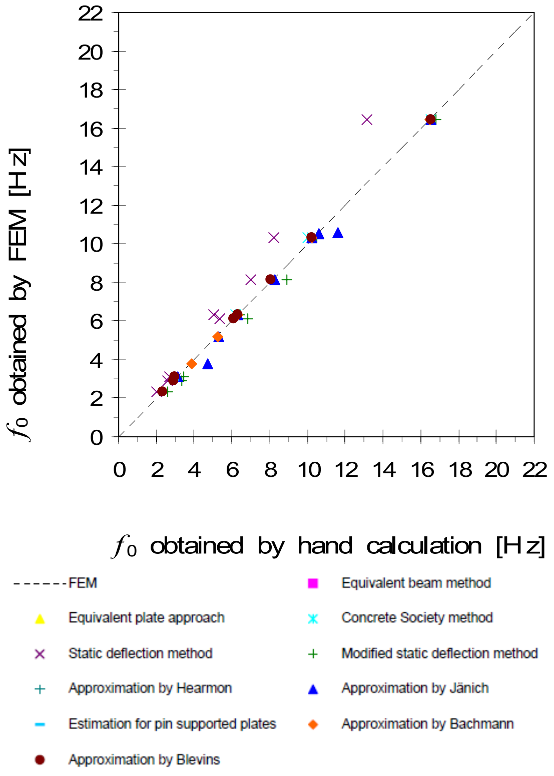

Figure 2 Comparison of hand-calculated and computed frequencies

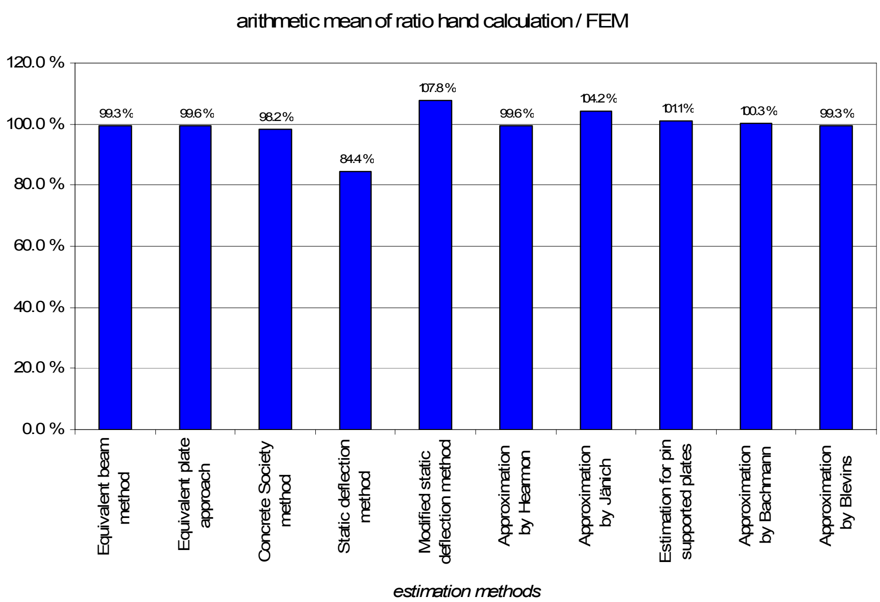

Figure 3 Average accuracy of approximations

The average ratio of hand calculated values to computed values is plotted for each respective estimation method in figure 3. It indicates the common high accuracy of all methods while also confirming the relative inaccuracy of the static deflection and modified static deflection methods.

This inaccuracy occurs as a result of these methods' general application. In contrast to all other tested approximations specified for one fixed set-up, both the deflection methods may be used for all kinds of boundary condition. An even more important factor for their less accuracy is their derivation from the equation of an one-degree-of-freedom system considering just one mass. However, even with variations of 15.6% and 7.8%, the estimated values still can be used for a rough prediction of vibration performance.

COBIAX FLAT SLAB

Cobiax flat slabs are a special solution in the lightweight flat slab system sector. They consists of hollow plastic spheres which are placed between the upper and lower static reinforcement of the slab as shown in figure 4. The spheres replace the concrete on the area with its lowest benefit. The slab maintains the same load bearing behaviour as traditional solid slabs and brings along some additional improvements.

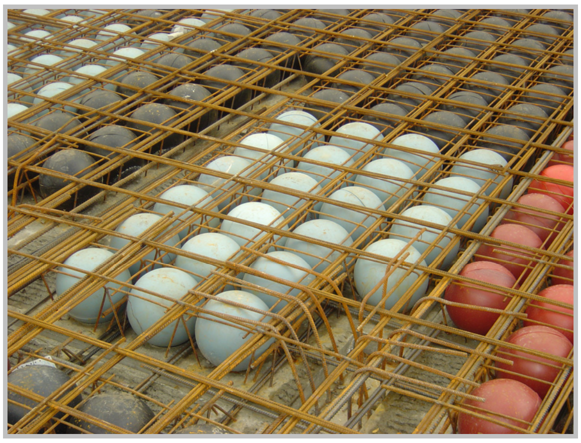

Figure 4 Cobiax flat slab module

Obviously, Cobiax flat slabs decrease their self-weight significantly. Depending on slab and ball dimensions a Cobiax flat slab weighs up to 35% less than a solid slab of equivalent geometry. This has a positive effect on the number of necessary vertical bearing elements (up to 40% less column usage) and results in savings of material for further structural elements like foundations. Other advantages include reductions in CO_2 emissions, savings in the amount of reinforcement needed, the application of all common standard designs and a smooth bottom view compared to common hollow concrete slabs.

In addition to the reduced mass the used hollow spheres have in impact of other qualities. Caused by the reduced moment of inertia, Cobiax flat slabs have a decreased stiffness compared to a solid slab.
This change affects some structural performances like the deflection or the vibration behaviour.

SPECIFIC NATURAL FREQUENCY OF COBIAX FLAT SLABS

Natural frequency is one of the fundamental parameters used in the determination of a structure's response to dynamic loads.
The the lowest natural frequency, the fundamental frequency for the simplest model of a dynamic system which has only one degree of freedom and no damping is given by:

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

In this case the natural frequency simply depends on the stiffness k and the total mass m of the system. This equation indicates the importance of these two qualities for a dynamic system. In case of Cobiax flat slabs these two parameters are decreased and have an opposite effect on the fundamental frequency; one will improve the solution and the other will impair it. The question is how much these characteristics impact on the product's performance and so which is to be the decisive factor.

To evaluate the influence of these parameters a detailed investigation was carried out. Seven common set-ups including

- Simply supported slab, one-way spanning
- Simply supported slab, two-way spanning
- 2 span slab, two-way spanning
- 3 span slab, two-way spanning
- 1x1 bay slab, supported by columns
- 2x1 bay slab, supported by columns
- 3x3 bay slab, supported by columns

were regarded. Additionally the dimensions and the loading were changed for each slab

In terms of Cobiax qualities three different sphere diameters (22.5cm, 31.5cm and 45cm) for slab thicknesses of 30cm, 40cm and 60cm were used.

In total 470 different floors were investigated to ensure the consideration of a wide field of application.
The fundamental frequency of each floor was calculated with finite element software and considered Cobiax flat slabs' properties as well as conventional solid slabs.
Knowledge about fundamental frequencies of both systems allows a comparison of the slab types and an assessment of Cobiax flat slabs' vibration qualities.

RESULTS

Every single slab considered in this investigation showed higher natural frequencies for Cobiax flat slabs. Their frequencies range between 3.6% and 11.9% higher than their solid equivalents.

But as presented in figure 5 it is noticeable that for all three curves the predominance of Cobiax slabs decreases with incremental loading.

With help of extrapolations the expected intersection of each curve and the x-axis can be estimated. This will provide a 'critical' loading at which Cobiax slabs will achieve the same natural frequencies as solid slabs. Every additional loading would change the ratio and the solid slabs would gain higher natural frequencies.

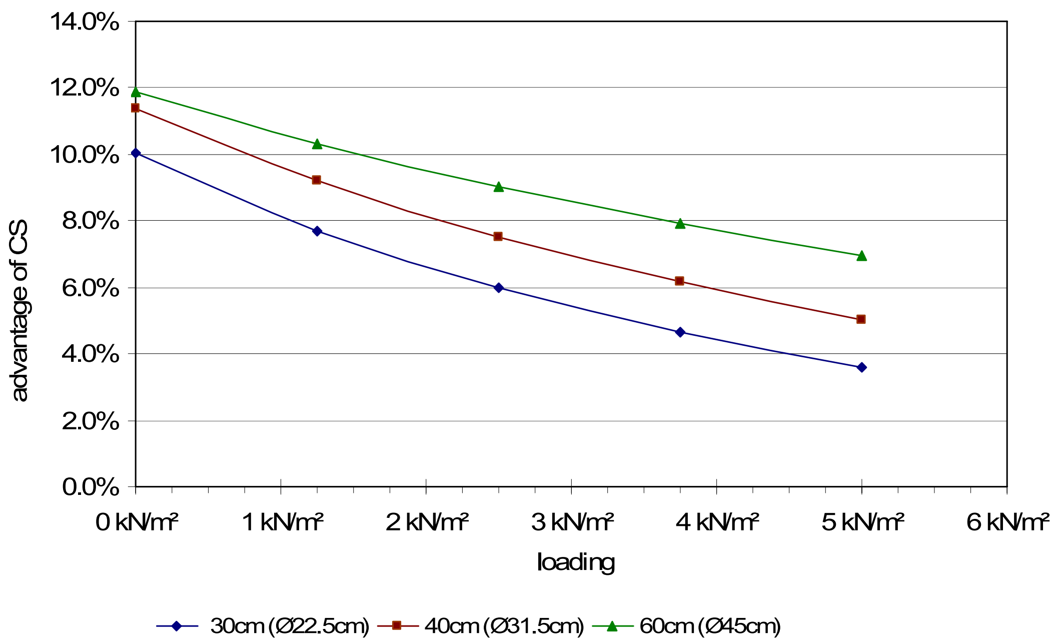

Figure 5: Cobiax advantages against loading

The obtained 'critical' values for each of the three slab/ball-combination are:

30cm / 22.5cm	max. q = 15.1 kN/m²
40cm / 31.5cm	max. q = 12.2 kN/m²
60cm / 45cm	max. q = 16.8 kN/m²

The change at these is caused by the decreasing relevance the Cobiax slabs' reduced self-weight. If the applied loadings are similar to the self-weight, the reduction of approximately 30% mass is an advantage for Cobiax slabs. However, once the applied loads increase, the self-weight becomes just a small amount of the overall load and is therefore negligible. If this occurs, the only difference on the part of Cobiax slabs is the reduced stiffness which has a negative impact on the vibration behaviour and therefore leads to lower natural frequencies.

CONCLUSION

In terms of simplified hand calculation methods is was shown that even if some variations occurred among each method, all methods provided good predictions and suffice for an initial assessment.

In case of the specific behaviour of Cobiax flat slabs according to floor vibration the overall conclusion is that they possess higher fundamental frequencies for all investigated combinations compared to conventional solid slabs.
However, it was also presented that this only occurs if a specific load to self-weight ratio exists. Indeed, Cobiax slabs lose their advantage of mass reduction after a certain point; but due to the amount of loadings required to realise this, its general performance is not affected.
More precisely, the usual fields of application for Cobiax flat slabs are residencies, offices, public buildings and car parks which could all be expected to bear a typically lesser imposed load than the 'critical' values.

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